

Shadow in the Galactic Center: Theoretical Concept – Prediction – Realization

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Abstract

General Relativity (GR) was created in November 1915, and since its creation this theory has undergone many tests. The first realistic cosmological models were proposed in the works of Friedmann, written in the 1920s. For a long time, Friedmann's cosmological works were actually banned in the Soviet Union due to philosophical reasons, since the models where the birth and evolution of the Universe occurs were considered ideologically unacceptable. Due to great achievements in relativity and cosmology and due to increasing interest in these branches of science in the last decades, we recall the development of relativistic astrophysics and contribution of Russian researchers to these studies. Since one of the world leaders in physical cosmology A. A. Friedmann passed away in September 1925, it is reasonable to outline the main achievements of physical cosmology over the past 100 years. We also discuss observational and theoretical achievements in confirmations of relativistic observational predictions for black holes, including the closest supermassive black hole in our Galactic Center. We outline the evolution of black hole shadow from the purely theoretical concept to observable quantities for supermassive black holes in Sgr A* and M87*.

Keywords: foundations of GR, cosmology, supermassive black holes, Galactic Center, M87*, synchrotron radiation, VLBI observations

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1. 110 years of success of GR development

General Relativity (GR) was developed by A. Einstein after intensive conversations with D. Hilbert in November 1915 [1–5]. In spite of difficulties to create a consistent quantum gravity in numerous attempts made by different authors [6–11], classical GR passed all possible tests at different scales.

In 1917, a truly revolutionary event in cosmological studies took place since the Universe started to be a subject for studies not only by philosophers but also by physicists [12]. In this year A. Einstein obtained the first cosmological model of static Universe based on his theory of relativity [13]. In this paper, Einstein assumed that the spatial distribution of matter

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in the Universe is uniformly isotropic and homogeneous (now it is called the cosmological principle). Many consequent researchers used the principle in their studies after him. Later, A. Eddington proved that Einstein's solution for static Universe is instable [14]; therefore, such a solution for static Universe cannot be realized in nature. To construct a static Universe model, A. Einstein introduced a cosmological constant, since a gravitational attraction must be compensated by a repulsion which is created by the cosmological constant. Many people repeated after G. Gamow [15, 16] that Einstein admitted to Gamow that introducing the λ term was the biggest Einstein's blunder; however, most likely it was a legend invented by Gamov [17], see also an essay on the book [18]. In spite of these arguments, some researchers think it was a real story [19–21], but taking into account the personalities of Gamow and Einstein, Livio's argumentation seems convincing.

The first realistic cosmological models were considered by Russian mathematician Alexander Friedmann in the 1920s [22, 23] (English translations of these fundamental articles were published in [24, 25]). Similar results were obtained by Belgian Abbe George Lemaître [29], where a derivation of the law which describes velocities of distant objects as dependence on distances toward them was observationally discovered by E. Hubble, and later this law began to bear his name [30, 31]². Later, this Lemaître's paper was translated into English and published in MNRAS, which was the main astronomical journal at that time [34] (see also republication of the paper [35] in the original French edition with comments by J.-P. Luminet [36]). Perhaps, J. Peebles was the first author [37] who noted that the Hubble law was indeed derived by Lemaître in 1927, and Peebles emphasized the issue in the conference talk devoted to 50 years since the creation of Big Bang cosmology proposed by G. Lemaître in [38] (see also republication of this paper [39] with comments by J.-P. Luminet [40]).

Since the derivation of the Hubble law in the English version of Lemaître'a paper was omitted [41–43], some people assumed that perhaps it was a censorship case, but after conversations with MNRAS editors M. Livio got documents [44] that Lemaître himself prepared the English translation of his original French text and he intentionally omitted his analytical derivation of the Hubble law and his estimation of the Hubble constant, since in 1929 Hubble obtained the Hubble law from observations [30]. D. Block proposed to apply Stigler's law for the Hubble law case [45]³.

At the end of 1932, Robert Milliken invited Lemaître to visit Caltech. On January 11, 1933, Lemaître was invited to deliver a lecture at the Mount Wilson Observatory, where E. Hubble worked. A. Einstein followed this Lemaître's lecture. When journalists asked Einstein about his impression of Lemaître's cosmological model, Einstein replied, "This is the most beautiful

¹Interesting comments on these Friedmann's papers were published dedicated to jubilees of the first Friedmann's paper on cosmology [26–28].

²In the 1920s, Lemaître spent a few years in the USA, worked at the Harvard Observatory, defended his PhD in MIT in 1927, personally knew Vesto Slipher, who analyzed Doppler spectral shifts and found that distant nebulae are preferably moving from us [32]. E. Hubble used Slipher's data on velocities of distant nebulae, and now some historians of science proposed to rename the Hubble constant [33]. However, we should note that an evaluation of distance is a more complicated problem in comparison with velocity estimates using Doppler shift measurements.

³Stigler's law was introduced by S. Stigler and it was motivated by Robert K. Merton's work on the reward system of science [46]. The simplest form of the law tell us, "No scientific discovery is named after its original discoverer" [47]. Very similar ideas were expressed by V. I. Arnold. He wrote, "The Arnold Principle. If a notion bears a personal name, then this name is not the name of the discoverer. The Berry Principle. The Arnold Principle is applicable to itself" [48]. Really, as we know, Arnold's principle was formulated earlier by Merton and Stigler.

and satisfactory explanation of creation to which I have ever listened!" [49]. Soon after that Duncan Aikman published an interview with G. Lemaître, where he claimed that there is no contradiction between religion and science and they provide two different ways to truth [50]. This position of Lemaître and Einstein was in contradiction with the official position of Soviet ideologists and, despite the fact that Einstein was highly valued by officials of the Soviet Union, Einstein's position on philosophical issues was criticized. Thus, in the introduction "On ideological vices in the book "The Evolution of Physics" by A. Einstein and L. Infeld", written by S. G. Suvorov and placed before the Russian translation made by him [51] (it is very fine that the book was translated into Russian), Suvorov instructively informed readers about Einstein's mistakes and, in particular, noted that "one of the judicial errors of the authors of the book is the incorrect interpretation of the development trends of modern physics". Suvorov was the head of the Science Department in the Central Committee of the Communist Party of the Soviet Union and later a deputy chief editor of the main Soviet physical journal "Soviet Physics" Uspekhi"; however, he was not a working physicist who had achieved significant scientific results, but he thought (perhaps due to his high position in Soviet establishment) that he had a right to teach one of the greatest scientists how to interpret trends in the development of physics.

In 1936, Soviet astronomer M. Eigenson⁴ from the Main Astronomical Observatory (Pulkovo) wrote a book [53] where he presented the Soviet official point of view on cosmology, and this position was also expressed in an article [54] written by A. L. Zelmanov (in the article, Eigenson's book was cited among other important references). According to opinions of Soviet astronomers, the Universe exists in infinite space and for infinite time, while expanding Universe theories proposed by Friedmann, Lemaître, Gamow, et al. are not allowed according to Soviet ideological point of view. It was a key difference between Soviet and Western cosmological schools [55, 56]. It means that, in contrast to other branches of physics, for thirty years Soviet cosmologists were not protected by "the nuclear shield" against ignorant criticism of modern physical theories, as was discussed by V. P. Vizgin [57]. The ban on the expanding models was from the 1930s to 1963, when the Soviet Academy of Sciences celebrated 75 years since A. A. Friedmann's birthday.

It would be reasonable to recall the cases of administrative pressure on Soviet cosmologists in the 1950s. In 1955, the Soviet Academy of Sciences' Department of Physics and Mathematics organized a special session devoted to the 50th anniversary of the (special) relativity theory, where cosmological talks were delivered by leading Soviet theorists. Instructor of the Central Committee of the Communist Party of the Soviet Union (CC CPSU) A. S. Monin⁵ and the head of the Science Department of CC CPSU V. A. Kirillin wrote a letter to the Presidium of the Soviet Academy of Sciences where they strongly criticized the Session where achievements of relativistic cosmology were discussed. The authors of the letter thought that the talks of

⁴I. S. Shklovsky wrote that M. Eigenson was the Communist Party Secretary in the Pulkovo Observatory in the 1930s [52].

⁵Monin was Kolmogorov's PhD student, developed Kolmogorov's turbulent model for atmospherical diffusion, defended his PhD thesis on the subject in 1949 and thesis for the degree of the Doctor of Sciences in 1956. Monin was the director of the Oceanology Institute of the Soviet Academy of Sciences for more than 20 years (from 1965 to 1987) and he was elected a full member of the Russian Academy of Sciences in 2000. In 1958, A. S. Monin wrote a letter to the Secretariat of the Central Committee of the Communist Party of the Soviet Union on forthcoming elections in the Academy of Sciences (https://ihst.ru/projects/sohist/papers/mp/1994/ilizarov.htm). It seems unlikely that Monin did not understand the truly highest scientific level of such scientists as Landau, Tamm, Zeldovich, Lifshitz and others; therefore, apparently, the letter with such assessments of scientific reports and works of the aforementioned scientists, who created the glory for Soviet science, was caused by opportunistic considerations.

E. M. Lifshits, Ya. B. Zeldovich, L. D. Landau and V. L. Ginzburg had significant disadvantages and were not criticized properly by the audience [58]. At the beginning of the letter, it was written, "... The program of the session, prepared by the Organizing Committee, which included Academicians Tamm and Landau, Corresponding Member of the USSR Academy of Sciences Ginzburg and Professor Lifshitz, was unsatisfactory, as it included reports by Academician Landau, Corresponding Member of the USSR Academy of Sciences Ginzburg and Professor Lifshitz, who do not work in the field of relativity theory and are known for their nihilistic attitude to the development of methodological issues of this theory..." Time has passed, and now we know the importance of the criticized scientists for Soviet theoretical physics, and it is hard to believe that their colleagues criticized them, since both Monin and Kirillin later became great Soviet organizers of science and full academicians in the field of physics and its applications.

In the 1940s, G. Gamow proposed his version of the hot Universe model where he estimated a helium nucleosynthesis in primordial Universe [59, 60], and soon after that his students calculated Cosmic Microwave Background (CMB) radiation [61] which should have now temperature around 5 K. In consequent calculations the CMB temperature was slightly different, but always it was several kelvins. The presence of CMB temperature was considered as an important feature of the hot Universe model proposed by Gamow. To mark Einstein's 70th birthday, the American Physical Society has published a collection of papers by famous physicists, including articles by Lemaître [62] and Gamow [63], in the special issue of Reviews of Modern Physics (https://journals.aps.org/rmp/issues/21/3), and Lemaître's and Gamow's papers (by both Georges) were published on adjacent pages. In 1948, a steady-state model for Universe evolution was proposed by H. Bondi, T. Gold and F. Hoyle [64, 65] in addition to the Friedmann-Lemaître-Gamow model discussed above. To explain the Hubble law, these authors of the steady-state model assume that matter may be continuously created throughout space and time. Some outstanding astronomers (F. Hoyle, J. Narlikar, G. Burbidge and some others) supported this point of view until the end of the 20th century [66] in spite of strong criticism of their opponents. In 2014, historians of science translated Einstein's unpublished notes written in 1931 and found that in these notes Einstein considered a cosmological model where he used ideas which were very similar to ideas expressed by Bondi, Gold, Hoyle and their followers [67]. As "Nature" columnist Davide Castelyecchi noted, perhaps steady-state model critics would not have been so aggressive if they had known that Einstein could once have held the same point of view [68].

Since the information from the Soviet encyclopedia was considered the absolute truth, was not discussed and criticized, it was accepted that encyclopedia statements are something like a system of axioms that must be followed by any Soviet citizen (including scientists). The article on cosmology [54] clearly stated that the Friedmann–Lemaître model was unscientific and was being promoted by the West for ideological and religious reasons. The de facto ban on the consideration of non-static Friedmann–Lemaître models of the Universe had a negative impact on the development of theoretical and observational research in the Soviet Union. For instance, let us recall the discovery of CMB radiation in the Soviet Union. As is known, the CMB radiation (which is one of the signatures of the hot Universe model developed by G. Gamow in the 1940s and 1950s) was discovered by T. Shmaonov at the Pulkovo Observatory several years before A. Penzias and R. Wilson (who were awarded the Nobel prize in 1978) [69]. For

⁶In 1940, Andrew McKellar, analyzing the spectra CN and CH, found that interstellar medium was very cold with a temperature of approximately 2 K, or more precisely 2.7, 2.1 and 0.8 K [70]. However, astronomers did not have a correct explanation for this phenomenon for decades.

instance, there is a description of this Shmaonov's discovery in a fundamental book [71] (see also a review [72]). Shmaonov's supervisor was Professor S. E. Khaikin, who was one of the first L. I. Mandelstamm's students in Moscow, a great expert in theoretical physics, radiophysics, radio astronomy, and the dean of the Physics Department at Moscow State University. Khaikin was fired from Moscow State University, MEPhI, and Lebedev Physical Institute after being accused of promoting idealism and Machism. A nice book [73] was recently published about this remarkable scientist and teacher. Shmaonov asked Khaikin about possible interpretation of his discovery, and Khaikin replied that he has no explanation, but it has to be published because it may be very important [73]. In the 1950s, Soviet physicists did not cite Gamow's papers at all, Khaikin worked as the head of the Radioastronomy Department at the Main Astronomical Observatory in Pulkovo, Gamow was one of the brightest representatives of Leningrad school of physics, and there is no doubt Gamow's works were known to leading Soviet physicists, such as Khaikin, but he preferred not to declare that his student (Shmaonov) had received confirmation of Gamow's predictions. After defending his PhD thesis, T. Shmaonov worked at the Institute of General Physics in the laser physics laboratory headed by A. M. Prokhorov, who got the Nobel prize in 1965 for the laser discovery and was Academician-Secretary of the Department of General Physics and Astronomy (1973–1993). In 1978, when A. Penzias and R. Wilson got their Nobel prize for CMB discovery, A. M. Prokhorov criticized Shmaonov that he did not inform properly scientific community about his CMB studies and did not promote his discovery.

Sometimes people say that Shmaonov's achievements were not known for many years, since they did not have an appropriate cosmological interpretation. There is a popular opinion that no one in the Soviet Union knew about the Gamow model of the hot Universe and the predictions of this model. However, this interpretation does not look correct. As mentioned earlier, in the Soviet Union the dynamic models of the Universe (including Gamow's models) were considered inadequate descriptions of the Universe and their consideration was not welcomed by official ideology and philosophy. In addition, despite the fact that Gamow was one of the most famous Soviet theoretical physicists, he did not return from a scientific trip abroad without the permission of the authorities. Thus, the mention of Gamow's works could be interpreted as a support for his disloyal attitude towards the Soviet government⁷.

Therefore, even if some experts understood that Shmaonov's achievements had a cosmological interpretation, they preferred not to demonstrate their understanding in order to avoid the danger of being condemned for supporting the provisions of physical cosmology, which were criticized by Soviet philosophers. In 1963, apparently, Soviet authorities decided to reconsider the assessment of Friedmann's cosmological works, and the Soviet Academy of Sciences held a session of the Department of Physical and Mathematical Sciences dedicated to the 75th anniversary of Friedmann's birth. In particular, P. L. Kapitsa said in his speech [75] that "Friedmann's name has so far been undeservedly forgotten. This is unfair and it needs to be fixed. We must perpetuate this name. After all, Friedmann is one of the pioneers of Soviet physics, a scientist who made a great contribution to domestic and world science." In July 1963, the journal "Soviet Physics Uspekhi" published a special issue of the journal dedicated to the 75th anniversary of A. A. Friedmann, which contains articles by famous scientists such as P. Ya. Polubarinova—Kochina, V. A. Fock, Ya. B. Zeldovich, E. M. Lifshitz and I. M. Khalatnikov, as well as Russian translations of Friedmann's articles on cosmology published in German in 1922 and

⁷In his interview, Gamow said to Ch. Weiner on April 25 and 26, 1968 (Gamow died on August 19, 1968), "...You see, my situation with Russian scientists is that physicists and astronomers know that I am persona non grata, and they are afraid to write to me, and I don't want to write to them because I bring them into trouble. But biologists don't…" [74].

1924. Moreover, in his review, Zeldovich started quoting Gamow's papers (earlier the Soviet researcher did not mention Gamow's papers). Initially, Zeldovich criticized Gamow's papers on the hot Universe model [76] in spite of that Soviet researchers did not mention Gamow and his papers after 1932, when Gamow did not return to the Soviet Union from a foreign trip, but Zeldovich immediately recognized the hot Universe model as a correct cosmological approach after the CMB discovery by A. Penzias and R. Wilson. Thus, it can be said that in 1963 the Soviet Union lifted the ban on discussing realistic cosmological models that consider the origin and evolution of the Universe. To mark the centenary of Friedmann's birth in 1988, the Soviet Academy of Sciences held a representative conference on gravity in Leningrad, which was attended by leading world experts in gravity and cosmology. A remarkable scientific biography was published [77] (soon after that the English translation of the book was published [78]), where, in particular, the authors wrote, "He discards a centuries-old tradition, which a priori, prior to all experience, considered the Universe eternal and forever motionless. He is making a real scientific revolution. Just as Copernicus forced the Earth to revolve around the Sun, so Friedmann forced the Universe to expand". At that time the Soviet Union stopped to be an atheistic state and Soviet cosmology started to be a part of international science.

For simplicity reasons, until the end of the 1990s, theorists and astronomers thought that $\Lambda \approx 0$ and they adopted $\Lambda = 0$. However, even in 1965 E. B. Gliner considered cosmological (inflationary) models with accelerating expansion [79, 80] (see also an essay on the scientific life of this remarkable scientist and personality in [81]). Such Gliner's models looked very exotic and some outstanding Soviet cosmologists (including Ya. B. Zeldovich) criticized them; on the other hand, V. L. Ginzburg and A. D. Sakharov tried to support Gliner and his studies [81, 82]. However, Gliner's results were nearly forgotten for decades [81].

In 1998, analyzing clusters with z > 0.5, astronomers found that $\Omega_m = 0.2^{+0.3}_{-0.1}$, while flat models with $\Omega_m = 1$ are ruled out by these data since probabilities for flat models are $p < 10^{-6}$ [83] (Ω_m is a matter density in critical density units). Since at that time cosmologists believed that the cosmological Λ constant "naturally" must be vanishing, an outstanding expert Neta Bahcall stated that the Universe has a light weight and it must expand forever [84]⁸. Assuming that SNIa is a standard candle, astronomers found that the Universe is flat $(\Omega_m + \Omega_{\Lambda} = 1)$ and $\Omega_m = 0.28$ [85–87]. It means that $\Omega_{\Lambda} \approx 0.7$ and the Universe expands with an acceleration. For this remarkable discovery, Saul Perlmutter, Adam Riess and Brian Schmidt got a Nobel prize in 2011. Before the discovery of the vanishing Λ term, it was in the left-hand side of Einstein equations and was treated as a geometrical part of the equations which characterizes Riemann tensor, while the expression on the right corresponds to the stress-energy-momentum content of spacetime; however, soon after the discovery M. Turner suggested to remove the Λ term to the right-hand side of the equations [88–90]. Therefore, the term could be treated as a part of the stress-energy tensor and, in principle, Λ may be not a constant but a function of spacetime and it was stated to name dark energy (at that time M. Turner had a financial support from the US Department of Energy in Fermi Lab and the University of Chicago). Consequent observations of CMB, galaxy clusters and distant supernova data confirmed conclusions on the presence of Λ term (dark energy). Constraints on Ω_m , Ω_{Λ} , Ω_k from these data are given in [91].

In March 2025, the DESI (Dark Energy Spectroscopic Instrument) collaboration found that it is possible that dark energy is different from an ordinary Λ constant [92]. It means that the

⁸She declared that the Universe is open in her inaugural article as a new member of the National Academy of Sciences elected on April 29, 1997. The article was published one year after her election and the achievement was interpreted one of the most significant in astronomy and cosmology in the first part of 1998.

cosmological model may be different from the standard Λ CDM approach; therefore, theorists should consider advanced cosmological models which are beyond the standard one.

2. GR tests

In 1942, Eisntein's assistant Peter Bergmann published one of the first books on general relativity [93] where he mentioned only three experiments that confirmed GR predictions; namely, he discussed the Mercury anomaly, light deflection in observations of foreground stars near the solar disk during a solar eclipse (which were evaluated by Einstein in November 1915) and gravitational redshifts of spectral lines for white dwarfs. In spite of the fact that observations of stars near the solar disk during the solar eclipse generally confirmed GR prediction firstly on May 29, 1919 [94], the accuracy of results was not very good and some scientists were rather skeptical [95]. Therefore, astronomers tried to check these results again and again with an approving precision (a detailed description of this expedition with a discussion of related issues was given in book [96] published 100 years after these remarkable observations). For instance, former Einstein's assistant Erwin Freundlich⁹, after analysis of his observations during the solar eclipse in Sumatra on May 9, 1929, concluded that "(1) A deflection exists; (2) It is not Newton's; (3) It seems to be greater than Einstein's" [99]. Freundlich found that in average the deflection angle is around 2".2 instead of 1".75. However, Eddington was at the Meeting and expressed his skepticism when he said, "I should like to congratulate Professor Freundlich on the cessation of the bad luck that has dogged him for 15 years. He was the first to make the attempt; now he has at last met the success, he found interesting — almost too interesting results. He has at least shown that there is a case for further investigation. I find it difficult to believe that 1".75 can be wrong. Light is a strange thing, and we must recognise that we do not know as much about it as we thought we did in 1919, but I should be very surprised if it is strange as all that".

Bergmann also noted that for the two first effects the theoretical magnitude of the effect only slightly exceeds the magnitude of the experimental errors, and he concluded that the quantitative agreement of experiments with general relativity remains questionable. In a more recent review and book [100, 101], C. Will presented a much more longer list for confirmations of GR predictions for different astronomical objects; a more popular but more fresh presentation is given in [102].

3. Bright stars as test bodies to test gravity at the Galactic Center

In order to determine the gravitational field in the area that interests us, it is necessary to follow the trajectories of the test particles. For example, Kepler, analyzing the movements of the planets in the Solar System, formulated laws that later became known as Kepler's laws, and Hooke and Newton discovered the law of universal gravitation, which described the action of gravity everywhere, not just in the Solar System. The nearest supermassive black hole is located at the center of our Galaxy. In order to study the gravitational field in the vicinity of the center of our Galaxy and, in particular, to test the hypothesis about the presence of a black hole there, we can conduct observations of the motion of bright stars there. In his Nobel lecture, R. Genzel outlined a way to an evidence that there is a supermassive black hole in the Galactic Center [103]. In the 1960s–1980s, there was an intensive discussion concerning the existence

⁹His attempts to measure displacements of foreground stars to test GR predictions were described in [97] (see also a nice popular book [98]).

of supermassive black holes and an energy release in quasars and AGNs, in particular, the presence of supermassive black hole in the Galactic Center. In 1969, D. Lynden-Bell proposed the binding energy of black holes as a source of an energy release in quasars [104]. Soon after that, J. Bardeen noted that an energy release could be higher for Kerr black holes [105]. D. Lynden-Bell and M. Rees considered different mechanisms for an energy release in the Galactic Center [106]. However, some researchers thought that the black hole mass in GC cannot be so high; for instance, Soviet astrophysicist L. M. Ozernoi and his group from the Lebedev Physical Institute of the Soviet Academy of Sciences insisted that models of supermassive black hole in the Galactic Center should be ruled out [107, 108]. However, astronomers led by Ch. Townes (University of California, Berkeley), analyzing results of observations for interstellar clouds, concluded there has to be a compact mass around $(2-4)\cdot 10^6 M_{\odot}$ in the Galactic Center [109– 113. British theorist M. Rees also argued that the black hole mass in the GC should be around $5 \cdot 10^6 M_{\odot}$ [114], meanwhile D. A. Allen and R. H. Sanders thought that the black hole mass should be around 100 M_{\odot} [115]. Different opinions concerning the black hole mass were a good stimulus to find new arguments to convince opponents. In 1987, R. Genzel and C. Townes presented a black hole mass estimate in the range $2-3\cdot 10^6 M_{\odot}$ and concluded that the evidence for supermassive black hole mass $(M \approx 10^6 M_{\odot})$ is substantial but not convincing [116].

Using diffraction-limited "speckle" imagery starting in 1991/1992 on the 3.5 m New Technology Telescope (NTT) of the European Southern Observatory (ESO) in La Silla/Chile, MPE astronomers found proper motions of stars as close as 0.1" from Sgr A*, which actually coincides with our Galactic Center (GC) [117, 118]. Since 1995 Ghez's group (University of California, Los Angeles) started to use the 10-m Keck telescope to monitor the bright stars near the Galactic Center, and the authors obtained similar results [119]. In 2002, the MPE group started to work with the 8.2 m Very Large Telescope (VLT) on Paranal, and the authors declared that they found a star (S2) which was approaching Sgr A* at 10-20 mas, and its period was estimated as 15.2 years [120] and the black hole mass was estimated as $M(0) = 3.7 \cdot 10^6 M_{\odot}$. Since the S2 (S02) star orbit is highly elliptical (e = 0.88) with a peri-distance of 14 mas (17 light hours or 1400 R_S (R_S is the Schwarzschild radius for supermassive black hole at the GC), for $M(0) = 4.26 \cdot 10^6 M_{\odot}$ according to the corrected estimates [121]), the orbit is suitable to test GR effects. Similar eatimates for orbital parameters of the S02 star are given by the Keck group [122]. Monitoring its trajectory gives an opportunity to test gravitational effects in the GC, as was done by the Keck and VLT groups. Even preliminary results on the trajectory of the S2 star gave an opportunity to constrain extended mass distribution [123] and parameters of stellar cluster and dark matter distribution near the GC [124]. At the ESO workshop held in Garching, April 4–8, 2005, it was proposed to combine four VLT telescopes in an interferometer (General Relativity Analysis via VLT InTerferometry, or GRAVITY), but proceedings of the workshop were published only 3 years after the Workshop date [125, 126]. The GRAVITY facilities were designed and made by a French-German-Portuguese Consortium of 6 Institutes (plus ESO) and installed on Paranal in July 2015 [121]. A description of the GRAVITY interferometer was done in [127]. Currently there are plans to extend baseline and create a VLTI interferomer with a baseline about a few kilometers [128] where key components will be VLTs and the GRAVITY interferometer. In August 2012, the IAU congress was held in Beijing and the Chinese journal "Research in Astronomy and Astrophysics" published a series of reviews on the most pressing issues in astronomy by leading experts. In particular, M. R. Morris, L. Meyer and A. M. Ghez published a review on observations of the Galactic Center in different bands and the interpretation of these observations [129]. Commenting on observational results of both groups [130, 131], R. Genzel claimed [121] that "the gravitational potential indeed is dominated by a point mass, whose position is identical within a mas uncertainty with that of the radio source Sgr A*". In May 2018, the star S2 was at the apocenter of its orbit, a fairly good approximation of which is an ellipse centered at Sgr A*. One of the well-known tests of general relativity is the gravitational redshift of spectral lines and the effect is strongest in the strongest gravitational field which is near the apocenter. The GRAVITY collaboration measured the gravitational redshifts and confirmed GR predictions [132–134]; in 2019 the Keck group published its results on gravitational redshifts of the S2 star near the apocenter [135].

Due to a slow progress in studies of dark matter and dark energy puzzles, extended theories of gravity have been proposed to try to explain these puzzles as gravitational phenomena [136– 139. Sometimes, f(R)-theories of gravity do not have a Newtonian limit in a weak gravitational field approximation, and the Solar System data put severe constraints on these theories [140]. Observations of the S2 star trajectory also gave an opportunity to constrain parameters of these theories [141, 142]. In the 2010s, it became clear that the accuracy of observations of the orbits of bright stars does not yet allow us to verify the predictions of the General Relativity; nevertheless, it is possible to limit the parameters of alternative theories of gravity that have been actively discussed in recent years. Thus, in the paper [143], restrictions on the parameters of the Yukawa theory were obtained and later, in the works [144, 145], restrictions on the graviton mass were obtained, and this restriction is comparable with the restrictions obtained from gravitational-wave experiments done by the LIGO-Virgo-KAGRA (LVK) collaboration [146, 147], and the best mass constraint $m_g < 1.76 \cdot 10^{-23}$ eV was obtained by the LVK collaboration from analysis of O1–O3a runs. Later, our constraints of graviton mass done from observations of bright stars near the Galactic Center were improved by Hees et al. using new Keck group's data [148–150]. Our constraints on graviton mass together with other bounds done from other observations are presented in PDG table for graviton mass bounds [151]. Further improvement of graviton mass bounds with observations of bright stars near the GC were found in [152–154]. In the paper [155], it was shown that there is an opportunity to constrain a tidal charge of black hole from analysis of bright star trajectories. The conventional model for the Galactic Center with a supermassive black hole and some alternative models were discussed in [156, 157]. Analyzing trajectories of bright stars, the GRAVITY collaboration considered constraints on parameters of alternative models for the GC [158]. In 2020, the GRAVITY collaboration detected the Schwarzschild precession for the S2 star orbit [159] (it was the second GR effect confirming GR predictions for the Galactic Center). Using the GRAVITY results on the Schwarzschild precession, parameters of extended theories of gravity were constrained in [160], while an influence of a bulk distribution of matter on the precession was discussed in [161]. Using observational data obtained by the GRAVITY collaboration, constraints of Yukawa gravity parameters were found in [162]. Taking into account the Schwarzschild precession found by the GRAVITY collaboration, graviton mass bounds were found in [163, 164]. Recently, new bounds on the fifth force parameters were discussed from observational data obtained by the GRAVITY collaboration [165].

It is generally believed that the Galactic Center contains a supermassive black hole surrounded by a small mass spatial distribution of matter. However, Ruffini, Argüelles and Rueda proposed replacing the supermassive black hole with fairly dense dark matter [166] (later this approach started to be called the RAR model). In the simplest approximation, we have that a dark matter density in core is a constant. A few years ago, Becerra-Vergara et al. claimed that the RAR model provided a better fit for trajectories of bright stars in the Galactic Center [167]. Really, as was shown in papers [168, 169], in the framework of the RAR model, stars move along elliptical trajectories since there is a harmonic oscillator potential inside a ball with a constant

density, and in this model the centers of these ellipses coincide with the origin of coordinates (or, in other words, with the center of a sphere of constant density), while observations done by the Keck group and GRAVITY collaborations showed that, in the first approximation, the stars move in a Newtonian potential and the center of symmetry of the gravitational potential coincides with the focus of the ellipse; therefore, a ball with a constant density cannot be a suitable substitution for supermassive black hole in the Galactic Center.

Analyzing orbits of bright stars, the GRAVITY collaboration obtained constraints on bulk matter distributions inside orbits of bound bright stars [170–172]. A comprehensive review of observational tools for studying black holes is presented in the article [173].

4. Black hole shadow in brief

In 1973, James Maxwell Bardeen discussed a thought experiment that suggested that there was a glowing screen behind a black hole. In this case, the observer will see a small dark spot (shadow) on the background of the screen [174]. It is a purely GR effect since there are no shadows for massive point in Newtonian gravity. For a long time, this model was not applied to astrophysical black holes, since astronomy does not have a bright screen behind a black hole, and for a long time in our Galaxy, stellar-mass black holes were mainly considered, whose shadow size is about a million times smaller than that of the shadow itself, the size in the center of the Galaxy (GC). In addition, questions were raised about the possibility of distinguishing a shadow from a dim astronomical object. Using estimates of the expected parameters of interferometers to be created in the near future and of those updated about 20 years ago, which turned out to be fairly accurate estimates of the mass of the black hole in the GC, Zakharov et al. [175] predicted the possibility of reconstructing the shadow in the center of the Galaxy using a ground-based or space-based interferometer operating in the millimeter or submillimeter range (also in this work, for the first time, the capabilities of the Millimetron interferometer for such needs were mentioned). Our prediction became true in May 2022, when the Event Horizon Telescope (EHT) collaboration presented the results of reconstructing the shadow of the center of our Galaxy (the shadow of the supermassive black hole in M87 was reconstructed in 2019). These reconstructions were based on the EHT observations made in 2017, so the analysis of observational data for M87* was carried out for more than 2 years, and for Sgr A* for more than 5 years. For the Reissner-Nordström metric, the authors [176] obtained analytical expressions for the shadow size as a function of charge, and later these results were generalized to the case of tidal charge [177, 178]. These results were presented and promoted at different conferences [179–182]. We were discussing the possibilities of estimating the parameters of alternative theories of gravity using shadow size estimates made by the EHT collaboration, in particular, based on these observations, the tidal charge can be estimated [183, 184]. We are also discussing the possibilities of using the Millimetron equipment for shadow reconstruction in the M87* and Sgr A*. In our recent study [185–187], we discussed the formation of shadows in cases where naked singularities or wormholes replace black holes in the centers of galaxies and considered the cases where generalizations of Kerr black hole metrics have no shadows.

5. VLBI ideas

Ideas of VLBI (Very-Long-Baseline Interferometry) were proposed by Leonid Ivanovich Matveenko at the beginning of the 1960s. Matveenko recalled the initial stage of development of this concept in [188–190]. Ten years after the start of development of this concept L. I. Matveenko wrote an article with the title "A radio telescope the size of a Globe" [191] in

one of the most widely read popular sciences magazines "Science and Life" in the Soviet Union (for example, the number of copies of issue 10 in 1973 was 2,900 thousand). Really, science was very popular in the USSR at that time. This slogan was so nice that many people used it after Matveenko, usually forgetting to specify the author who came up with and used this slogan for the first time.

According to Matveenko's memoirs, in the 1960s he worked as a radio physicist at the Deep Space Network (DSN) Center near Yevpatoria (Crimea). The Center was established to control and communicate with Soviet spacecrafts. The Center had a radio interferometer with baseline length around 500 m. In 1962, Matveenko decided to increase a baseline and proposed the first interferometric experiment using telescopes DSN and near Simferopol. In autumn 1962, he reported about his proposal at a seminar of the Radio Laboratory (head V. V. Vitkevich) in Pushchino, and this proposal was not supported [188, 189]. Moreover, "V. V. Vitkevich excused that the Laboratory cannot support publishing the paper or carrying out an experiment" [189]. Matveenko requested a support from radio astronomers G. B. Sholomitskii and N. S. Kardashev at the Sternberg State Astronomical Institute (SSAI) of Moscow State University and reported these ideas at the institute seminar [192]. The institute director D. Ya. Martynov supported these ideas and recommended to take a patent. The head of the Radio Astronomy Department at SSAI was I. S. Shklovsky. In 1962, SSAI¹⁰ sent the VLBI proposal by L. I. Matveenko, N. S. Kardashev, and G. B. Sholomitskii to the Soviet Patent Bureau. However, experts in the Patent Bureau did not approve the proposal and noted that usually they issued a patent for a scientific or technical result but not a method [189]. Matveenko requested the Patent Bureau to give a patent or a permission for publication in a journal, and finally the paper was published [195] almost 2.5 years after the first application for patent. However, the authors' proposal on the possibility of using ground-based space interferometry was removed from the article at the request of the reviewer. Subsequently, this idea was given in a popular article [191] (on the 20th anniversary of the idea, Matveenko's article was published in a popular journal |197|).

In summer 1963, the director of the Jodrell Bank Radio Observatory Sir Bernard Lovell visited the Soviet Union as a special guest of the Soviet Academy of Sciences (at that time the Observatory had the largest radio telescope in the world, and the telescope could track the first Soviet Sputnik [198]). Shklovsky invited Matveenko to present his ideas on long-range interferometry in front of the distinguished guest¹¹. Lovell approved Matveenko's idea, but expressed doubt that there are powerful, practically point-like astronomical sources for the observation of which such an angular resolution is necessary. However, we have to remind that in 1963 quasars were discovered which are very compact and bright and the VLBI facilities started to be very useful for observations of these objects.

¹⁰In his notes on the development of radio astronomy in the Soviet Union, I. S. Shklovsky noted the importance of VLBI techniques [193, 194], developed by N. S. Kardashev, L. I. Matveenko and G. B. Sholomitskii. In these notes, Shklovsky changed the order of the authors that was in the patent application and in the Soviet Radiophysics journal publication [195]. Shklovsky emphasized that these techniques are especially useful for observations of compact objects, for instance, quasars; however, the method of VLBI observations was not listed among significant achievements of the Radio Astronomy Department of SSAI [196].

¹¹Lovell wrote in his diaries that he felt some kind of sickness and he was unhappy after this visit, see https://luna.manchester.ac.uk/luna/servlet/detail/Manchester~14~14~1623~192985?qvq=q%3Abernard+lovell&mi=1&trs=5. Perhaps due to these circumstances the Soviet-British collaboration was not started as previously agreed in the Crimea in the summer of 1963 and it was written in a Memorandum concerning joint observations with interferometer between the radio telescopes in Yevpatoria and Jodrell Bank at $\lambda = 32$ cm [188].

Remembering the VLBI idea realization in the Soviet Union, L. I. Matveenko wrote, "Without going into the details of the formation and development of the VLBI, I can only note that the main problems in our country were not bureaucratic obstacles, but the "support" of colleagues [188]". Mathematician M. I. Monastyrsky (ITEP) expressed a very similar idea, analyzing several cases of outstanding Soviet mathematicians [199], "By the way, we (in Russia) love to lament about the underestimation of Russian scientists in the West. But a careful analysis of the real facts shows that the greatest obstacles to the recognition of Russian (Soviet) scientists are other Russian (Soviet) scientists."

In 1976, the first global telescope was created, or more precisely, on April 26 and May 6, 1976, radio telescopes in the European USSR, the United States, and Australia were linked together in order to observe H₂O maser sources with an angular resolution of 0.1 mas (milliarcsec) [200]. Four antennas formed this global telescope, namely, 22-m (Simeiz), 26-m Maryland Point Observatory of Naval Research Laboratory, 40-m Owens Valley Radio Observatory in California, and 64-m NASA telescope in Tidbinbilla (Tidb) in Australia.

In the 1970s, Matveenko participated in interferometric observations of compact radio sources (3C 84, 3C 273, 3C 345, NRAO 150) at $\lambda = 1.35$ cm, where radio astronomers used the following telescopes: 22-m Simeiz (USSR), 20-m Onsala (Sweden), 100-m Effelsberg (Germany), 37-m Haystack (Westford, Massachusetts, USA) [201].¹³ In 1976, the Crimea–Haystack interferometer observed the source W51 with maser lines at OH and H₂O at $\lambda = 1.35$ cm [203].

In the 1980s, L. I. Matveenko proposed a project with space—ground interferometry [204]. Later, the Japanese HALCA-VSOP and Russian Radioastron missions were launched and operated on the orbit. However, after processing the results of observations obtained with the help of these space radio telescopes, it seems impossible to assert that fundamental discoveries have been made that could be considered appropriate to the space cost of projects.

6. Synchrotron radiation in astrophysics

The Institute of Theoretical and Experimental Physics (ITEP) in Moscow was founded by Academician A. I. Alikhanov in December 1945, and the scientific community of the institute could have celebrated the 80th anniversary of its founding, but since 2022 the institute has ceased to exist. The head of the Theoretical Department at ITEP in the 1950s—1960s was a famous Soviet theorist I. Ya. Pomeranchuk. Brief essays on Pomeranchuk's life and his scientific achievements are presented in [205, 206]. He made a number of remarkable discoveries; in particular, he predicted the existence of electromagnetic emission from electrons moving in magnetic fields [207–209] (this physical phenomenon was analyzed earlier by G. A. Schott [210]; however, in the 1940s his studies were nearly forgotten). Since the 1940s rather powerful accelerators were in action [211], soon after publications of the papers by Pomeranchuk, Iwanenko and Artsimovich the emission was discovered at a synchrotron and it was started to be called a synchrotron emission. Really, the first detection of X-ray radiation from electrons in 70-MeV synchrotron has been done by Elder et al. [212, 213]. The history of the discovery is presented in [214–216]. According to I. I. Gurevich's opinion, this Pomeranchuk's work on magnetic

¹²Here we could remind A. Friedmann, whose works were considered relevant in the USSR, Gamow, whose works Soviet physicists preferred not to cite for reasons known to them, Matveenko, whose ideas were not supported by the leadership, Gliner, whose ideas were not supported by many influential physicists, despite the support of such leading theorists as Ginzburg and Sakharov.

¹³According to NASA ADS, L. I. Matveenko had eight joint papers with future Nobel prize winner R. Genzel in small groups of co-authors, see, for instance, results of maser line observations with the Crimea–Effelsberg and Haystack–Green Bank observations in November 1979 [202].

bremsstrahlung (synchrotron) radiation should have been crowned with a Nobel Prize [217] (I. I. Gurevich was an outstanding Soviet physicist, who made a significant contribution to the implementation of the Soviet Atomic Project).

One of the simplest explanations for the origin of synchrotron emission is given in Chapter 34 in the nice classical textbook (see, for instance, a reprinted edition [218]). A brief history of the discovery and development of synchrotron radiation was presented in [219] (and it was repeated in significant points in [220]). In these papers, the authors noted that a significant progress in understanding the synchrotron radiation was made after publications of papers by Alfred Lienard [221] and Emil Wiechert [222], where the authors introduced delayed potentials. Different applications of synchrotron radiation in science and technology are outlined in [223]. The first particle accelerator Tantalus, fully dedicated to synchrotron light experiments, started to operate in 1968 in Wisconsin; experiments which were done in the first years of its operation are outlined in [224]. A general description of such facilities of the first generation was given in [225].

In the mid-1940s, I. S. Shklovsky followed a talk delivered by Yu. B. Kobzarev. Kobzarev reported that radio physicists detected radio emission from the Sun. After analysis of the problem, Shklovsky concluded that radio emission could be generated by electrons in moving magnetic fields in the solar corona [226]. V. L. Ginzburg obtained a very similar conclusion in his first astronomical paper [227]. On May 20, 1947, a total solar eclipse was to occur in Brazil, and the Soviet Academy of Sciences decided to send an expedition to observe the solar eclipse in the radio and optical ranges. Papaleksi was appointed the leader of the expedition. He believed that observing the Sun during a solar eclipse would allow him to test theoretical predictions that radio emission was generated in the corona. But in February 1947, Papaleksi died and S. E. Khaikin was appointed the leader of the expedition. Two famous theorists (Shklovsky and Ginzburg), who predicted the existence of radio emission from the solar corona, participated in this expedition. Both of them left their memoirs about this exotic trip [52, 228]. Khaikin and Chikhachev performed this delicate experiment, in which not a telescope, but a ship tracked the position of the eclipsed Sun, and confirmed the predictions of theorists [73]. After that, using their radio observations of the Sun during the solar eclipse at the wavelength $\lambda = 1.5$ m, Khaikin and Chikhachev published papers where they described their discovery of radio emission from the solar corona [229, 230]. Only many years later this discovery was officially registered, and the Committee on Discoveries and Exploration under the Council of Ministers of the USSR registered in the State Register of the USSR on April 28, 1970 under No. 81 with priority of October 28, 1947 the discoveries by S. E. Khaikin, B. M. Chikhachev, Papaleksi D. N. and issued a diploma on September 14, 1971 on the discovery of radio emission from the solar corona [73] (it would be reasonable to remind that N. D. Papaleksi passed away on February 3, 1947, Khaikin died on July 30, 1968; therefore, these two remarkable scientists did not see an official state recognition of their remarkable discovery). Optical observations of stars near the solar disk during the total solar eclipse to test GR predictions were not made due to bad weather conditions.

The Crab nebula was discovered around two hundred years ago, and in the 1950s there were many observations, but a central engine mechanism was unknown. And in 1953 a famous Soviet astrophysicist I.S. Shklovsky proposed a synchrotron emission as a key mechanism to explain observational data in a wide range of frequency band from radio to optics [231]. Later, Shklovsky's model was confirmed with consequent X-ray data. In his memoirs, Shklovsky noted that the idea to use a synchrotron mechanism for the Crab nebula was among his brightest insights [193, 194]. Now we know that synchrotron radiation plays a key role in the generation

of electromagnetic radiation near many different astronomical objects, including pulsars and black holes.

7. Shadows in M87* and Sgr A*

As observational and experimental technologies advance, the possibilities for testing theoretical predictions of various theories, including Einstein's theory of relativity, increase. However, theorists are faced with important questions related to which objects should be observed, when, and with what observational facilities. As was noted earlier, Matveenko reported his VLBI ideas to a famous British radio astronomer Lovell in 1963, and the latter expressed skepticism regarding the need to build such facilities, since powerful compact radio sources were unknown; however, as Shklovsky noted, quasars, which are such sources, became known already in 1963. It was also mentioned that the first space—ground interferomer was proposed by L. I. Matveenko, and he promoted this idea in the 1980s and it was realized only by Japanese HALCA (Highly Advanced Laboratory for Communications and Astronomy), also known for the project name VSOP (VLBI Space Observatory Programme), which was launched in 1997 and officially stopped in 2007. In 2011, the Russian space—ground Radioastron mission started and it stopped operating in 2019. In spite of significant amount of observations with these facilities, breakthrough results were not obtained. The Japanese Space Agency cancelled the VSOP successor ASTRO-G (VSOP-2) due to the risk of not achieving significant scientific results.

In the 2000s, when a scientific program of Radioastron was in preparation, it was known that a black hole mass in the Galactic Center is around $4 \cdot 10^6 M_{\odot}$ and distance is around 8 kpc, the Schwarzschild radius size is around 10 μ as, while the angular resolution of ground-space telescope was expected to be around 8 μ as at the shortest wavelength $\lambda = 1.35$ cm. Since the angular resolution of such facilities was comparable with a typical scale of the GR effects, it would be reasonable to expect to find an interesting GR effect in observations of GC with Radioastron facilities. In 2000, Falcke et al. considered a toy model where they simulated an opportunity to observe a shadow around the black hole at GC in different wave band [232]. The authors concluded that there is a chance to observe a shadow at $\lambda = 0.6$ mm, while interstellar scattering practically destroyed shadows for $\lambda = 1.3$ mm. Using available estimates for the black hole mass $2.6 \cdot 10^6 M_{\odot}$ [117, 119], Falcke et al. predicted the angular diameter of the shadow in Sgr A* from the GR calculations alone to be around $(30 \pm 7) \mu as$ [232]. The angular resolution of the VLBI interferometer as the Event Horizon Telescope has now is around 25 μ as; therefore, perspectives to observe a shadow for the Galactic Center did not look very optimistic, and Falcke et al. mentioned that perhaps it would be necessary to use a wavelength below 0.2 mm or even to use MAXIM interferometer to solve such a problem [232] (developed by W. Cash, for details see [233, 234], but unfortunately, the MAXIM project was cancelled around two decades ago due to high cost and lack of guaranteed sound results).

Due to the presence of secondary images near black hole shadows [235], we predicted that the black hole shadow at the Galactic Center could be reconstructed from VLBI observations in mm and sub-mm bands and in this case ground–space observations with Millimetron facilities will be very useful [175]. In April 2017, the EHT collaboration with a global VLBI network observed M87* and Sgr A* at 1.3 mm wavelength. In April 2019, the first publications on shadow reconstructions for M87* were done [236]. The authors adopted a distance toward this object as (16.8 ± 0.8) Mpc, evaluated a shadow diameter (42 ± 3) μ as and found that the black hole mass $M = (6.5 \pm 0.7) \cdot 10^9 M_{\odot}$. The authors declared that "the photons at 1.3 mm wavelength observed by the EHT collaboration are believed to be produced by synchrotron emission" [236]. Later, the EHT collaboration reported polarization distribution in the M87* ring around the

shadow [237] and magnetic field map distribution which is consistent with polarization [238]. The data strongly supported initial assumptions of the model that astronomers observed the ring due to synchrotron emission of electrons moving in magnetic fields around the black hole in M87*.

In May 2022, the EHT collaboration presented a shadow reconstruction for Sgr A* [239] after more than 5 years of data processing after observations in April 2017. The authors declared that the shadow diameter is $(51.8 \pm 2.3) \mu as$ (with 68% credible interval). This fact illustrates difficulties in the problem of shadow reconstruction for an object with strong variability on a scale of tens of minutes. However, this excellent result was a confirmation of our prediction, done in 2005 [175], that the black hole shadow for GC can be reconstructed from VLBI observations in mm band and the shadow diameter should be around 50 μ as (see also discussions of the issue in [240, 241]).

In 2024, the EHT collaboration reported polarization distribution in the ring near the black hole shadow in Sgr A* [242] and distribution of magnetic fields near the horizon of the black hole [243]. These observational data support a synchrotron radiation model for detected flux of electromagnetic radiation from the Galactic Center.

Thus, the concept of a black hole shadow has grown from a theoretical concept proposed by J. Bardeen [174] to the prediction that the size and shape of the shadow of a black hole in the Galactic Center can be obtained from VLBI observations in the millimeter or submillimeter range [175] and the reconstruction of the shadow by the collaboration in 2022 from their observations in April 2017 [239].

8. Are shadows 100% confirmations for presence of SMBHs in M87* and Sgr A*?

In popular literature and elsewhere, one can find the assertion that observation of the shadow of black holes and analysis of the trajectories of bright stars provide 100% evidence of the existence of black holes in these objects. If it is necessary to choose a suitable model from a small number of alternatives under consideration, then the galactic center model may be preferable to other models. Therefore, in mathematical terms, when explaining a natural phenomenon, proof of the uniqueness of the model used is usually not given, since one can usually only speak of the preference of one model over another. Below we give an example of a metric that differs from the Schwarzschild black hole metric, but which similarly describes the shadows of black holes.

It is known that the influence of spin on the deformation of the shadow of a black hole decreases as the position of a distant observer approaches the axis of rotation of the black hole and the position of the observer for the cases of M87* and Sgr A* is far from the equatorial plane, we will limit ourselves to the case of spherical symmetry of the metric, which will simplify our subsequent reasoning. Let us consider a class of spherically symmetric static metrics. Ideas of our analysis is very similar to Friedmann's consideration done in his first cosmological paper [22, 24].

Let us recall the expression for such a metric

$$ds^{2} = A(r)dt^{2} - A^{-1}(r)dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$
(1)

Below we measure the linear distances r in M (M is a mass of a compact object). From the spherical symmetry of the metric it follows that the motion occurs in one plane ($\theta = \pi/2$) and the angular momentum is conserved: $r^2 \frac{d\varphi}{d\tau} = L$ (for photon we have $\tau = \lambda$). Since the metric is static, the energy is conserved: $E = A(r) \frac{dt}{d\tau}$ (for photon, $\tau = \lambda$). Assume that A(r) > 0.

Let us test the metric done in Eq. (1) with photons ($ds^2 = 0$). We consider this case just for simplicity, but it is possible to use the same analysis for massive particles moving in the metric. Then, we obtain from Eq. (1)

$$A(r)\left(\frac{dt}{d\lambda}\right)^2 - A^{-1}(r)\left(\frac{dr}{d\lambda}\right)^2 + r^2\left(\frac{d\phi}{d\lambda}\right)^2 = 0,$$
 (2)

since

$$\left(\frac{dt}{d\lambda}\right)^2 = \frac{E^2}{A^2(r)},\tag{3}$$

we obtain

$$\left(\frac{dr}{d\lambda}\right)^2 = \left(\frac{E^2}{A(r)} - \frac{L^2}{r^2}\right) A(r),\tag{4}$$

or

$$\left(\frac{dr}{d\lambda}\right)^2 = \left(\frac{1}{A(r)} - \frac{b^2}{r^2}\right) \frac{A(r)}{E^2},\tag{5}$$

where b = L/E. Therefore, the photon motion can occur only in regions where

$$\frac{1}{A(r)} \geqslant \frac{b^2}{r^2},\tag{6}$$

or

$$B(r) \leqslant \frac{1}{h^2},\tag{7}$$

where $B(r) = A(r)/r^2$. If we define the function A(r) = 1-2/r for $r \ge 3$, while $A(r) = r^2/27$ for r < 3, then $B(r) = (1-2/r)/r^2$ for $r \ge 3$ and B(r) = 1/27 for r < 3. As we noted above, we could compare Eq. (6) from Friedmann paper [22] and Eq. (4) or (5) and our Figure 1 with the

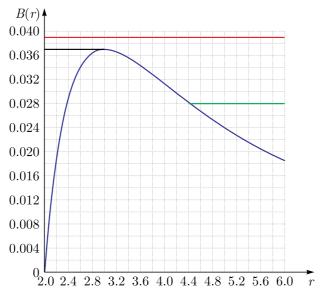


Figure 1. The blue curve represents $(1-2/r)/r^2$ for all r. The function B(r) corresponds to the blue curve for $r \ge 3$ and corresponds to the black horizontal straight line B(r) = 1/27 for r < 3. The red horizontal straight line b = 5.063 (B(r) = 0.039) corresponds to the capture of a photon; the green horizontal straight line b = 6 (B(r) = 0.028 for r > 4.4) corresponds to the case when the photon moves from infinity until $r \approx 4.4$, at this point the photon turns and moves to infinity again. The critical impact parameter $b = 3\sqrt{3}$ separates scatter and capture regions for the impact parameters. It is the same as for the Schwarzschild metric.

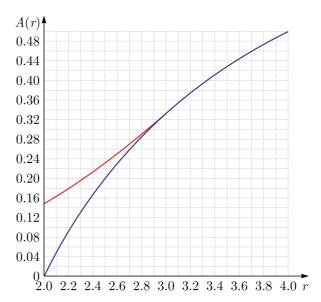


Figure 2. The blue curve represents the function A(r) for the Schwarzschild, while the red curve corresponds to our modified A(r). These red and blue curves are different only for r < 3.

figure in [22]. It is not strange since in both papers the authors were searching regions where some functions are positive.

If we conduct a thought experiment like Rutherford's experiment on the scattering of photons by a Schwarzschild black hole, we will not be able, even in principle, to distinguish the metric of a black hole from a metric where the function A(r) is replaced by a modified one in the interval 2 < r < 3 (see Fig. 2). Similarly, if we use bound orbits to test the black hole metric, the Schwarzschild metric will be indistinguishable from the modified metric considered above and done in Eq. (1), since the probe body tests gravity only in the range of radial coordinate periapsis to apoapsis, but in this region the metric in Eq. (1) coincides with the Schwarzschild one. We have considered only two ways to test the metric of a compact object, but even when considering other approaches there are limitations to the inference that the processing of observations leads to the conclusion that there is a Schwarzschild and/or Kerr black hole. It is probably more correct in this case to say that this black hole model describes the observational data best among the alternatives considered.

9. Conclusions

In this article, we recalled some fragments of the development of physics and astronomy in Russia; not all of them are well known even to specialists. The remarkable words of S. Weinberg about the history of physics come to mind. At the end of his life, this famous scientist worked on GR and astronomy and wrote several outstanding books on these subjects [244–247]. In one of his last popular books, Weinberg emphasized the importance of the history of science and wrote, "I am a physicist, not a historian, but over the years I have become increasingly fascinated by the history of science. It is an extraordinary story, one of the most interesting in human history. It is also a story in which scientists like myself have a personal stake. Today's research can be aided and illuminated by a knowledge of its past, and for some scientists knowledge of the history of science helps to motivate present work" [248].

Unfortunately, in Russia the achievements of domestic scientists were sometimes hushed up, thereby giving additional bonuses to foreign scientists. In this context, one can recall the

underestimation of the importance of Friedmann's work on cosmology, since Soviet philosophers and ideologists insisted that the Universe is infinite in time and space. On May 2, 1946, P. L. Kapitsa wrote a letter to Stalin, asking for support for the publication of Gumilevsky's book "Russian Engineers"; in particular, Kapitsa wrote [249], "It is clear from Gumilevsky's book that a large number of the largest engineering initiatives were born in our country, we ourselves were almost unable to develop them, often the failure to use innovation is due to the fact that we usually underestimated our own and overestimated foreign... Many of the organizational shortcomings still exist today, and one of the main ones is underestimating our own and overestimating foreign forces... After all, excessive modesty is an even greater disadvantage than excessive self-confidence... We will do this [develop national technology] successfully only when we believe in the talent of our scientist and engineer..." A number of measures were taken that led to the acceleration of the country's technological development, the social status of scientists was raised, but such ugly campaigns as the fight against sycophancy and cosmopolitanism also arose. Perhaps Kapitsa's letter to Stalin influenced his determination to begin the fight against servility to the West. Soviet writer and poet K. M. Simonov recalled that once at the meeting with writers Stalin said, "There is an issue that is very important and that writers need to be interested in. This is the theme of Soviet patriotism. If one takes our middle classes, scientific intelligentsia, professors and doctors, the sense of Soviet patriotism is not nurtured in them quite enough. They have an unjustified admiration of foreign culture. Everyone feels somewhat juvenile, not quite one hundred percent, they are used to considering themselves to be lifelong pupils... First the Germans, then the French — there was always this admiration of foreigners" [250].

As an example of the disgusting campaign to combat cosmopolitanism and servility in the 1940s and the 1950s, one can recall one of the days of V. L. Ginzburg, who later became the last Russian Nobel laureate in physics. On Ginzburg's birthday (October 4, 1947), the Soviet Literaturnaya Gazeta (Literature Newspaper) published an article by economist V. Nemchinov with a headline "Against servility!" [251]. The full text of this article can also be found in the anniversary album dedicated to the 100th anniversary of Ginzburg [252]¹⁴. Nemchinov noted in the article that, in his brochure on the atomic nucleus, Dr. Ginzburg does not refer to the Iwanenko-Gapon model of the nucleus; the author of the article writes that Ginzburg is hushing up this achievement of Soviet science. As for the lack of citation of the Iwanenko-Pomeranchuk article on synchrotron radiation, it is said that there is a completely absurd groveling before American science, since Ginzburg cited a review by an American author. There is nowhere to go beyond this shameful phenomenon of hushing up discoveries of Soviet science, of erasing Soviet authors. Once at the house of mutual friends, Ginzburg met a person (Schneiderman) who was the real author of the article signed by V. Nemchinov and in these conversations Schneiderman said that the authorities requested him to write an article criticizing Lysenko's opponents, basically geneticists, including A. R. Zhebrak. Iwanenko worked in TSKhA, learned about the article being prepared and recommended that cases from other branches of science

¹⁴V. Nemchinov was an academician, one of the leaders among Soviet economists and statisticians. He promoted mathematical methods in economy and was the Rector of the Timiryazev Agricultural Academy (TSKhA) in 1947. Since he defended genetics and TSKhA scientists working in this area of biology at the August Session of the Lenin All-Union Academy of Agricultural Sciences (VASKhNIL) in 1948 [253], soon after the session he was resigned from the rector position. See also a detailed report on the event in "On the Situation in Biological Science. Verbal Report Session of the Lenin All-Union Academy of Agricultural Sciences" [254]; practically, it was a defeat of genetics, after which it could not recover for several decades. As P. Pringle noted, neither the VASKhNIL founder and first president N. I. Vavilov, nor the biological institutes that Vavilov founded and led [255] (N. I. Vavilov died in prison in 1943) were mentioned at the session.

be included to reach a greater generality. So, V. L. Ginzburg was sure that Nemchinov's article was initiated by D. D. Iwanenko; however, in 1947 V. L. Ginzburg started to work in the Soviet Atomic Project¹⁵ and, according to his opinion, only these circumstances influenced the fact that he was not subjected to repression, since he was married to a repressed woman, as a cosmopolitan and a Jew [261]. On the same day, October 4, 1947, the plenum of the Higher Attestation Commission decided not to approve Ginzburg's academic title of professor [261] (two "gifts" for one birthday).

We outlined individual stages of development of research in the theory of gravity, in particular, in cosmology. In September this year it will be possible to sum up the results of 100 years of development of physical cosmology after the departure of A. A. Friedmann, and in November it will be time to sum up the results of development of GR for 110 years after its creation by A. Einstein.

Coming to the conclusion of the discussion of tests of general relativity, in particular, the new test associated with the reconstruction of shadows of black holes from observations, it can be noted that several years have passed since the reconstruction of the black hole shadow in M87* and the Galactic Center was presented. The images of the rings around these shadows have sometimes even been called images of black holes. However, the main thing in these images is the dark region inside the rings, since it is this region that characterizes the gravity of the black hole. One of the leaders of the EHT collaboration wrote a popular book [262] (in the Russian edition of the book, printed in 2024, in contrast to the picture at the envelope of the book in German, English and Italian editions, however, the publishers changed the EHT ring image for M87* with an artist view picture which wrongly reflects a gravitational field near a black hole, thus removing the main essence of the book and the author activity for decades) and paper [263] describing the way to reconstruct a bright ring around darkness.

We described how the purely theoretical shadow concept was transformed into the observational quantity which can be obtained from observations as the EHT collaboration demonstrated. As R. Genzel concluded in his Nobel prize lecture, any plausible astronomical model for the Galactic Center must include a supermassive black hole with a mass around $4 \cdot 10^6 M_{\odot}$ [103].

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Conflicts of Interest

The author declares no conflicts of interest.

 $^{^{15}}$ Ginzburg's key contribution to the creation of the hydrogen bomb is described in an article by one of the leaders of the Soviet Atomic Project, Yu. B. Khariton [256, 257] (see also [258–260]).

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