NATURAL SCIENCE REVIEW **1** 4 (2024)
nsr.jinr.int

Computational testing of independent component analysis for linear optics measurements at the NICA Booster

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Abstract

The accelerator complex NICA is at the stage of assembling and commissioning. A series of successful runs at the injection complex were carried out using various types of ions. It is planned to continue the linear optics measurements at booster synchrotron, for which several methods are considered. The first one is based on the analysis of turn-by-turn data of the beam orbit going from beam position monitors. The independent component analysis is used for the data processing and results to computation of betatron and synchrotron tunes, beta-functions, phase advances and dispersions. Other methods use orbit response matrix measured with alternate kicks by dipole correctors. Accuracy of optics restoration depends on the technical feasibility of betatron tunes and orbit measurements. Various methods should be firstly accommodated to the accelerator and tested using computational model in order to conclude their potentials and form requirements for future experiments with the beam. The paper describes implementation of independent component analysis to the computer model of the NICA Booster.

Keywords: synchrotron, linear optics, betatron function, computer simulation, measurements

1. Introduction

The accelerator complex NICA (JINR, Dubna, Russia) includes two heavy ion synchrotrons and a two-ring collider [1]. The main task for the accelerator team is boosting the maximal beam intensity in the collider. In particular, it is necessary to optimize the beam transmission and acceleration that include optics measurements and corrections. Linear optics measurements are planned at the first stage of the commissioning and their result should be expressed in the betatron tunes, beta-functions and dispersions. To solve such tasks, well-known methods implemented successfully in other accelerator centers are used. Published materials about the theory supporting the methods are available in [2]. There is no experience in carrying out such measurements at the accelerators of the NICA complex. Therefore, our initial goal is a preliminary analysis of the applicability of the methods to the NICA accelerators.

The Booster, being the first circular accelerator in the chain, accelerates light and heavy ions up to an energy of 570 MeV/nucleon. The Booster's circumference is 211 m. 24 two-coordinate Beam Position Monitors (BPMs) are used for the detection of the beam orbit which can be corrected by 24 two-coordinate dipole correctors. Booster quadrupoles are powered

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Received: 11 November 2024; Revised: 4 December 2024; Accepted: 16 December 2024; Published: 20 December 2024

from common power supplies that do not allow one to vary fields in each lens independently. Given this limitation, the analysis of turn-by-turn beam positions for all available BPMs based on the Independent Component Analysis (ICA) looks as a preferable method for linear optics measurements.

ICA [3, 4] has been applied many times to calculation of betatron functions of synchrotrons using turn-by-turn beam positions measured by the BPMs [5–7]. Transverse oscillations of the beam can be excited by a kicker or by the beam injection error. All the BPMs of a ring are used for providing turn-by-turn data for further calculation of betatron functions and tunes, as well as the betatron phase advances, and dispersion functions. The duration of the beam oscillations is typically determined by decoherence due to the tune chromaticity and the tune dependence on the betatron amplitudes.

Methods of beta-functions calculation based on usage of the orbit response matrix require iterative procedure for approximation of measured data by response matrices calculated using the computer model of the accelerator [8, 9]. The most known technique is LOCO (Linear Optics from Closed Orbits) [10] that provides not only restoration of linear optics, but also detection of the system errors leading to such optics. In this study, our task comes down to linear optics measurement, but not searching for causes of optics distortions. Therefore, LOCO is not discussed. Note also that the data acquisition for the turn-by-turn measurements is much faster than the time required for measurements of orbit response matrix which represent serious practical advantage.

2. Independent component analysis

ICA is a computational method used in signal processing for decomposition of multisource signal into independent signals. In accelerator physics, ICA is applied for separation of physical signals (betatron and synchrotron motions, various periodical crosstalk and noises) using measurements of beam transverse oscillations detected by the BPMs. Let the beam make N turns in a ring. The beam diagnostic system includes M BPMs. In this case, the measured matrix of orbit coordinates at all turns has the dimension $N \times M$. Orbit data at each BPM can be presented as a column $\vec{X}_m = (x_{1,m},\, x_{2,m},\, \ldots,\, x_{N,m})$, $m = 1, 2, \ldots, M$. The physics of betatron and synchrotron motions determines that each column data can be presented as a combination of all signals representing the physical state of the oscillating system with addition spurious signals and noise. Note that the systematic errors in BPM response may also distort the signals. Consequently, turn-by-turn data at BPM m is the composition:

$$\begin{array}{l} x_{1,m} = (s_{1,1} \cdot a_{m,1} + s_{1,2} \cdot a_{m,2} + \ldots + s_{1,M} \cdot a_{m,M})\,(1 + g_m) + \delta_m + \mathrm{Noise}_{1,m} \\ x_{2,m} = (s_{2,1} \cdot a_{m,1} + s_{2,2} \cdot a_{m,2} + \ldots + s_{2,M} \cdot a_{m,M})\,(1 + g_m) + \delta_m + \mathrm{Noise}_{2,m} \\ \ldots \\ x_{N,m} = (s_{N,1} \cdot a_{m,1} + s_{N,2} \cdot a_{m,2} + \ldots + s_{N,M} \cdot a_{m,M})\,(1 + g_m) + \delta_m + \mathrm{Noise}_{N,m} \end{array},$$

and the beam orbit represented by each BPM at turn n is

$$\begin{array}{l} x_{n,1} = (s_{n,1} \cdot a_{1,1} + s_{n,2} \cdot a_{1,2} + \ldots + s_{n,M} \cdot a_{1,M})\,(1 + g_1) + \delta_1 + \mathrm{Noise}_{n,1} \\ x_{n,2} = (s_{n,1} \cdot a_{2,1} + s_{n,2} \cdot a_{2,2} + \ldots + s_{n,M} \cdot a_{2,M})\,(1 + g_2) + \delta_2 + \mathrm{Noise}_{n,2} \\ \ldots \\ x_{n,M} = (s_{n,1} \cdot a_{M,1} + s_{n,2} \cdot a_{M,2} + \ldots + s_{n,M} \cdot a_{M,M})\,(1 + g_M) + \delta_M + \mathrm{Noise}_{n,M} \end{array},$$

where $\vec{S}_m = (s_{1,m},\, s_{2,m},\, \ldots,\, s_{N,m})$ — periodical physical signals describing the system state, g_m is addition to orbit reading by BPM m due to an error in its differential response (gain) which usually has the value up to several percent, δ_m is orbit offset at BPM m, $\boldsymbol{A} \in \mathbb{R}^{M\times M}$ is a mixing matrix of signals $\boldsymbol{S}$. As will be seen below, fraction of these signals describes actual beam motion, while other are due to noise, spurious signals and nonlinearity in the BPM gain. Thus, the measured matrix of orbit coordinates at all turns $\boldsymbol{X}$ can be presented by a product of matrices:

$$\boldsymbol{X} = \boldsymbol{S} \cdot \boldsymbol{A}. \tag{1}$$

The mathematical task is to define matrix $\boldsymbol{S}$, providing all oscillation modes of the considered system. Every column $\vec{S}_m$ of matrix $\boldsymbol{S}$ presents a single oscillation mode.

Each betatron motion mode in matrix $\boldsymbol{S}$ is presented by two orthogonal signals (sine and cosine) because each BPM sees different phases of betatron oscillation. Synchrotron motion, being a slow periodic process, at fixed turn appears at each BPM identically, so that longitudinal motion in $\boldsymbol{S}$ is presented by a single signal. Therefore, matrix $\boldsymbol{S}$ contains five basic modes: pairwise betatron and single synchrotron. If there are other periodic components in the system, they will appear in matrix $\boldsymbol{S}$: fast — by pair, slow — by single mode. So, $\boldsymbol{S}$ can be viewed as a temporal projection of the considered system.

Matrix $\boldsymbol{A}$ represents a spatial pattern of oscillation modes along the ring, i.e. their amplitude distribution along the ring. It allows one to calculate the betatron functions and the betatron phase advances. The spatial mode presenting the synchrotron motion is proportional to the dispersion.

Matrices $\boldsymbol{A}$ and $\boldsymbol{S}$ can be defined using ICA under conditions of mutual independence and non-Gaussianity[1] of signals $\vec{S}_m$. In the presence of coupling, it is enough to have only horizontal turn-by-turn orbit data from the BPMs. The number of BPMs needs to be more than the required number of modes planned for calculations. Further, if not stated otherwise, the matrices have dimensions $\boldsymbol{S} \in \mathbb{R}^{N\times M}$ and $\boldsymbol{A} \in \mathbb{R}^{M\times N}$.

Various algorithms are used in a core of ICA. The first of them, FastICA [11], is the easiest for implementation and is based on using the gradient descent method. The most widespread method is the Blind Source Separation (BSS) algorithm of second order using simultaneous diagonalization of covariance matrices of measured data with Jacobi angles technique [6, 12]. In this study, BSS showed good separation of the modes.

The first step in the implementation of BSS is preparation of measured data which consists of data centering of $\boldsymbol{X}$ (providing zero mean value, i.e. zeroing δ_m); and the second step is the data decorrelation. Removing correlations from a matrix (whitening) represents such a linear transformation of the input data that converts them into new data, the covariance matrix of which is identity matrix, i.e. the correlation of the data in the matrix is totally removed. This procedure is called whitening, as it brings the initial data closer to white noise. For centered matrix $\boldsymbol{X} \in \mathbb{R}^{N\times M}$, covariance matrix $\boldsymbol{C} = \boldsymbol{X}^T\boldsymbol{X}$ is calculated. Then it is used for computation of matrix $\boldsymbol{E}$, consisting of eigen vectors of $\boldsymbol{C}$, and diagonal matrix $\boldsymbol{\lambda}$, diagonal elements of which are eigen values of matrix $\boldsymbol{C}$. Finally, decorrelated matrix $\tilde{\boldsymbol{X}}$ is calculated using input data $\boldsymbol{X}$ in accordance with

$$\tilde{\boldsymbol{X}} = \boldsymbol{V} \cdot \boldsymbol{X}, \tag{2a}$$

$$\boldsymbol{V} = \boldsymbol{\lambda}^{-1/2} \cdot \boldsymbol{E}^T, \tag{2b}$$

where the square root of matrix $\boldsymbol{\lambda}$ in (2a) is a new matrix, elements of which are square roots of $\boldsymbol{\lambda}$ matrix elements. After transformation of the initial data using (2a), matrix $\tilde{\boldsymbol{X}}$ is split

[1] In this context, the non-Gaussianity means that the value is not normally distributed.

into K submatrices $\tilde{\boldsymbol{X}}_k$ using row-wise splitting (turn-by-turn samples) with a finite step of $N/K \in (1; N)$, and matrices $\tilde{\boldsymbol{C}}_k$ are calculated as

$$\tilde{\boldsymbol{C}}_k = \tilde{\boldsymbol{X}}_k \cdot \tilde{\boldsymbol{X}}_{k+1}^T,$$
$$k = 1, 2, \ldots, K,$$
$$K \leqslant N,$$
$$\tilde{\boldsymbol{X}}_k \in \mathbb{R}^{\frac{N}{K} \times M}.$$

Then symmetrical matrices of the following form are computed:

$$\boldsymbol{R}_k = \frac{\tilde{\boldsymbol{C}}_k + \tilde{\boldsymbol{C}}_k^T}{2},$$

and, finally, we should define such orthogonal matrix $\boldsymbol{W}$ that will diagonalize simultaneously all the matrices $\boldsymbol{R}_k$. K is chosen to have better result for simultaneous diagonalization, that can be achieved only approximately, and can be $K = 1$. To calculate $\boldsymbol{W}$, the Jacobi angles technique is used [12, 13], which has algorithmic realizations in programming languages [14] and can be easily used as a solution to the described problems.

At the final stage, the mixed matrix for calculation of an original physical signal based on measured data is computed:

$$\boldsymbol{A}^{-1} = \boldsymbol{W}^T \cdot \boldsymbol{V},$$

and original signals are calculated as

$$\boldsymbol{S} = \boldsymbol{X} \cdot \boldsymbol{A}^{-1}.$$

Now we have matrix $\boldsymbol{S} \in \mathbb{R}^{N \times M}$, in which betatron oscillations are presented by pairwise columns. Synchrotron motion is calculated from a single column:

$$\left(\frac{dp}{p}\right)_n = b \cdot s_{n,m_s}, \tag{3}$$

where constant b is the scaling factor that can be defined when comparing measured results and calculated data using a computer model. m_s — the number of data column in $\boldsymbol{S}$, responsible for synchrotron motion. It is not possible to define beforehand columns numbering, presenting betatron oscillations, synchrotron motion and other physical modes in matrices $\boldsymbol{S}$ and $\boldsymbol{A}$. ICA orders restored signals in accordance with their relative power. Typically, these signals are in the first 6–10 restored modes. Ordering of the modes can be done after computation of their Fourier series.

Fourier analysis of columns of matrix $\boldsymbol{S}$ responsible for the betatron motion yields the betatron tunes. Corresponding columns of matrix $\boldsymbol{A} \in \mathbb{R}^{M \times M}$ contain information about betatron functions and phase advances and also about the dispersion function of the accelerator at azimuths where the BPMs are installed. Let us call the beta-functions $\beta 1$ and $\beta 2$ assuming the presence of coupling of transverse motions. $\beta 1$ and $\beta 2$ go to the standard βx and βy used for rings without coupling. The functions can be calculated as

$$\begin{aligned}
&\beta 1_m = c \cdot \left(a_{m,n\beta 1_1}^2 + a_{m,n\beta 1_2}^2\right), \quad \varphi 1_m = \tan^{-1}\left(\frac{a_{m,n\beta 1_1}}{a_{m,n\beta 1_2}}\right), \\
&\beta 2_m = c \cdot \left(a_{m,n\beta 2_1}^2 + a_{m,n\beta 2_2}^2\right), \quad \varphi 2_m = \tan^{-1}\left(\frac{a_{m,n\beta 2_1}}{a_{m,n\beta 2_2}}\right), \\
&D_m = d \cdot a_{m,ns}.
\end{aligned} \tag{4}$$

Here scaling factors c and d, responsible for amplitude of betatron oscillations (proportional to emittance) and for proportionality between dispersion and orbit displacement (proportional to momentum deviation), can be found when comparing measured results and calculated data using computer model. Indexes $n\beta 1_1$, $n\beta 1_2$, $n\beta 2_1$, $n\beta 2_2$, ns — columns numbers in matrix $\boldsymbol{A}$, presenting pairs of betatron modes and synchrotron motion. Note that $\phi 1$ and $\phi 2$ are not affected by errors in the gains of BPM's differential response.

3. Usage of ICA in measurements of the beam optics

Testing of the described algorithm can be performed at a computer model of the accelerator. For that, firstly turn-by-turn orbit position at each BPM should be generated. Then prepared orbit data are processed using ICA, and linear optics of the accelerator is computed.

First, we consider the beam motion in the absence of betatron motion decoherence. For the NICA Booster, the calculation procedure was the following. The computer model of the accelerator is prepared using code OptiMX [15]. The model and all the calculations are made taking into account coupling of transverse motions. The reason is to develop all the algorithms in a general case, and the NICA Booster includes electron cooling system with 2.7-m solenoid operating with field level up to 1.5 kG. The NICA Collider, for which the methods intended, also has electron cooling system and longitudinal magnetic field in each of its detectors. From the optical model, alpha-, beta-functions, betatron phase advances, and variables describing value of coupling are exported. Based on these data, the circulation of an ion (center mass of the beam) in the ring is simulated. At each turn, the orbit position is saved for each BPM. The ion motion is calculated using an algorithm based on transformation eigen vectors of transfer matrices [16, 17]. Ion position at turn n in phase-space in place where BPM m is located can be calculated as

$$\begin{bmatrix} x_{n,m} \\ x'_{n,m} \\ y_{n,m} \\ y'_{n,m} \end{bmatrix} = \mathrm{Re}\left(\sqrt{\varepsilon_1}\,(\vec{v}_1)_m\,\mathrm{e}^{-i\left(\psi_1+(\mu_1)_m+n\mu_{1R}\right)} + \sqrt{\varepsilon_2}\,(\vec{v}_2)_m\,\mathrm{e}^{-i\left(\psi_2+(\mu_2)_m+n\mu_{2R}\right)}\right) + \begin{bmatrix} Dx_m \\ 0 \\ Dy_m \\ 0 \end{bmatrix}\left(\frac{dp}{p}\right)_n, \quad (5)$$

where $\varepsilon_{1,2}$ are the beam transversal emittances, $\mu_{1,2}$ are the betatron phase advances in two transverse planes, $\psi_{1,2}$ are the initial phases of the ion, $\mu_{1R,2R}$ are the betatron phase advances corresponding to the full turn in the ring, Dx, Dy — dispersion functions, dp/p — momentum deviation, n – turn number. Eigen vectors $\vec{v}_1$ and $\vec{v}_2$ at the azimuth where BPM m is installed are defined as

$$(\vec{v}_1)_m = \begin{bmatrix} \sqrt{(\beta 1_x)_m} \\ -\dfrac{i\,(1-u)+(\alpha 1_x)_m}{\sqrt{(\beta 1_x)_m}} \\ \sqrt{(\beta 1_y)_m}\mathrm{e}^{i(\upsilon_1)_m} \\ -\dfrac{iu+(\alpha 1_y)_m}{\sqrt{(\beta 1_y)_m}}\mathrm{e}^{i(\upsilon_1)_m} \end{bmatrix}, \qquad (\vec{v}_2)_m = \begin{bmatrix} \sqrt{(\beta 2_x)_m}\mathrm{e}^{i(\upsilon_2)_m} \\ -\dfrac{iu+(\alpha 2_x)_m}{\sqrt{(\beta 2_x)_m}}\mathrm{e}^{i(\upsilon_2)_m} \\ \sqrt{(\beta 2_y)_m} \\ -\dfrac{i\,(1-u)+(\alpha 2_y)_m}{\sqrt{(\beta 2_y)_m}} \end{bmatrix}. \quad (6)$$

Generalized Twiss functions $\alpha 1_x$, $\beta 1_x$, $\alpha 1_y$, $\beta 1_y$, $\alpha 2_x$, $\beta 2_x$, $\alpha 2_y$, $\beta 2_y$ in (6) describe coupled transversal motion. In the absence of coupling, nonzero values have only $\alpha 1_x$, $\beta 1_x$ and $\alpha 2_y$, $\beta 2_y$, which coincide with the commonly used alpha- and beta-functions of horizontal and vertical motions. υ_1 and υ_2 are the relative phases of horizontal and vertical components corresponding to the eigen vectors. Constant u characterizes value of coupling [16]. All the functions in (6) are exported from OptiMX. It is assumed in (5) that all the functions do not depend on the turn number, which is justified if the number of measured turns and chromaticities is sufficiently small.

In this numerical study, to account for the longitudinal motion, it is sufficient to introduce turn-by-turn energy gain as

$$T_n = T_{n-1} + U_{rf} Ze \cos\left(\omega_{rf} t_n + \varphi_0\right), \tag{7}$$

where U_{rf} — amplitude of accelerating voltage, Ze — ion charge, $\omega_{rf} = f_{rf} 2\pi$ — accelerating frequency, φ_0 — initial RF phase of the ion. Relative momentum deviation, as known, can be obtained as $dp/p = (\gamma/\gamma + 1)\, dT/T$, where γ — Lorentz factor. Synchrotron oscillations are obtained from (7) if one takes account of the dependence of ion orbit length on the beam energy.

Using (5), motion of a single ion is simulated and its trajectory equates with coordinates of coherent oscillations of the beam that will be measured in the experiment. Strictly speaking, association of a single ion motion with the beam center of mass is not correct, but for such calculations with testing of ICA, this assumption is sufficient.

The data were computed for both horizontal and vertical planes, but only horizontal coordinates (only x in left part of (5)) were used. So, matrix with turn-by-turn horizontal orbit data $\boldsymbol{X} \in \mathbb{R}^{N \times M}$ is generated that corresponds to the measured orbit from horizontal BPMs. For the NICA Booster, the calculations were performed with coupling introduced by the solenoid of electron cooling system. After $\boldsymbol{X}$ prepared, it is appended by different noises and periodical crosstalk. Then whitening is performed according to (2) and ICA implemented in Python.

Coherent oscillations of the beam can be excited in the measurements by single kick using transverse kicker, or, in the case of longitudinal motion, small jump in the RF phase. Also, beam oscillations excited by injection errors and measured immediately after injection can be used in the calculations. The calculations produced for the above-described case without decoherence show that required decomposition of the signals appears (Figure 1), even in the case of significant coupling and the white noise added.

The first pair of extracted signals presents betatron oscillations for mode 1, the third mode — synchrotron motion, the next pair — betatron oscillations of mode 2. Other signals do not contain physics. Fourier analysis of the signals yields the betatron and synchrotron tunes that are in good agreement with the optics model used to generate data (Figure 2).

Results of restoration of betatron functions, momentum deviation due to synchrotron motion, and dispersion function are also in good agreement with target data computed in the optics model (Figure 3). Rms accuracy of betatron functions restoration reaches $2 \cdot 10^{-2}$ for the white noise with peak value of 10% of betatron motion and random errors in BPM gains up to 5%. Without errors in the gains, the accuracy improves to $3 \cdot 10^{-3}$. Horizontal dispersion is restored with an accuracy of $1 \cdot 10^{-2}$. The betatron function of second transversal plane ($\beta 2$) also can be restored using horizontal orbit data, but with an accuracy an order of magnitude worse. It is better for computation of $\beta 2$ to use vertical orbit data.

After adding to input data a spurious signal with 50-Hz frequency (EMI from power net), the accuracy of synchrotron mode restoration is changed significantly. This is due to the fact that the synchrotron tune in the NICA Booster is $\sim$ 80 Hz at injection (where the measurements are planned). So, the synchrotron tune value is close to the added crosstalk frequency. Temporal modes restored by ICA are shown in Figure 4. Components of longitudinal motion (mode 3)

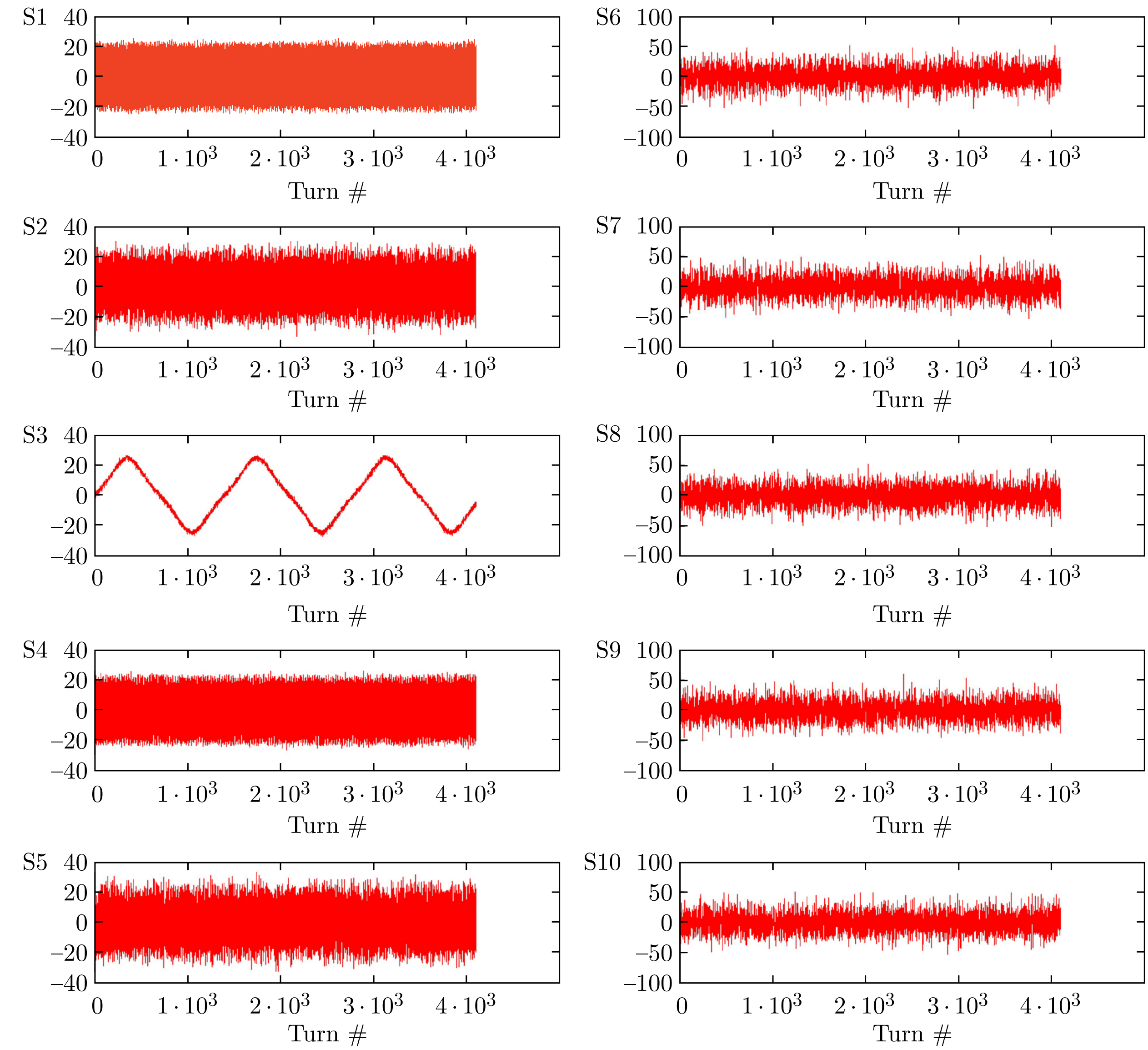

Figure 1. First ten temporal modes provided by ICA: components 1, 2 and 4, 5 represent betatron oscillations, component 3 corresponds to synchrotron motion, components 6–10 do not contain periodical states.

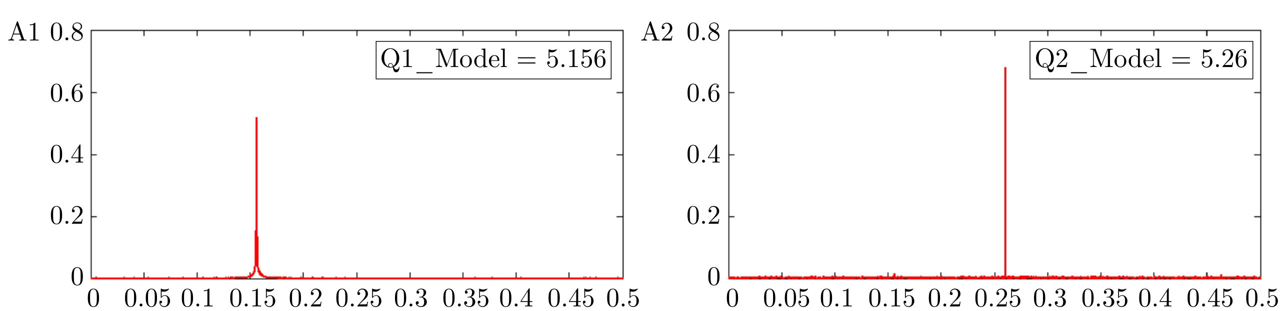

Figure 2. Results of Fourier analysis of betatron modes. Values in frames are targeted tunes.

and crosstalk (mode 5) are distorted strongly. Dispersion function is defined with a significant error (Figure 5, left) because of large inaccuracy of synchrotron oscillations selection (Figure 5, right). However, it weakly affects the accuracy of betatron functions restoration.

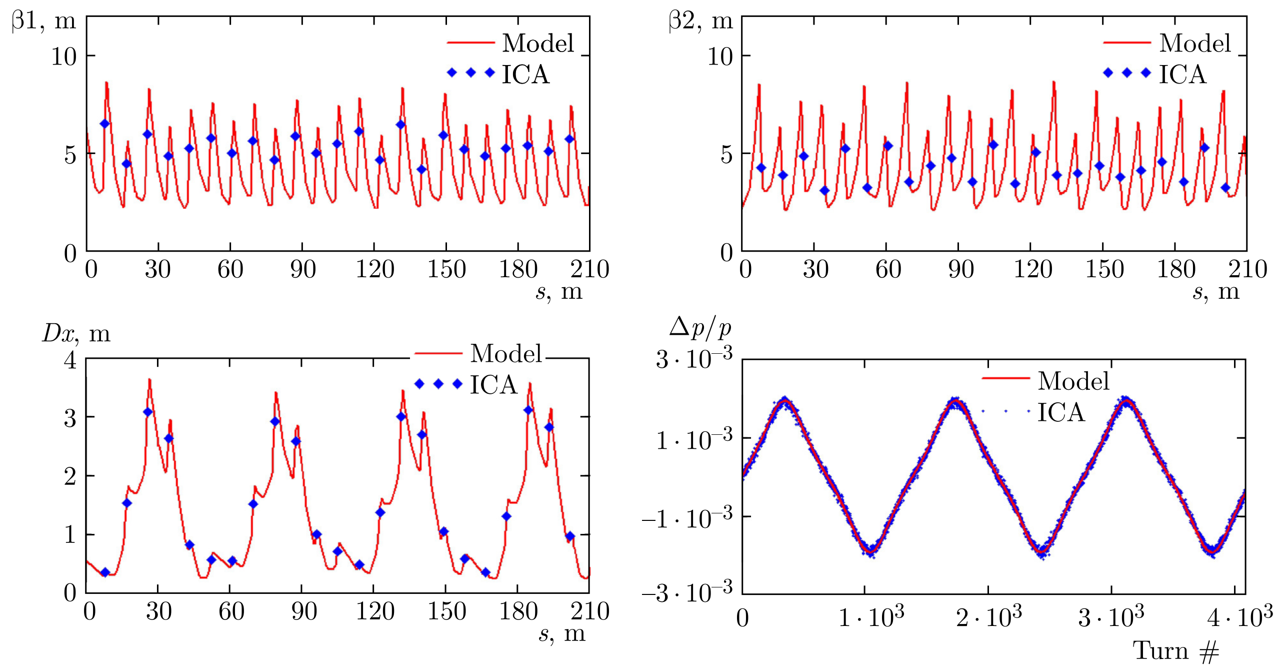

Figure 3. ICA results for restoration of betatron functions, horizontal dispersion, and synchrotron motion. Horizontal orbit was used for the calculations.

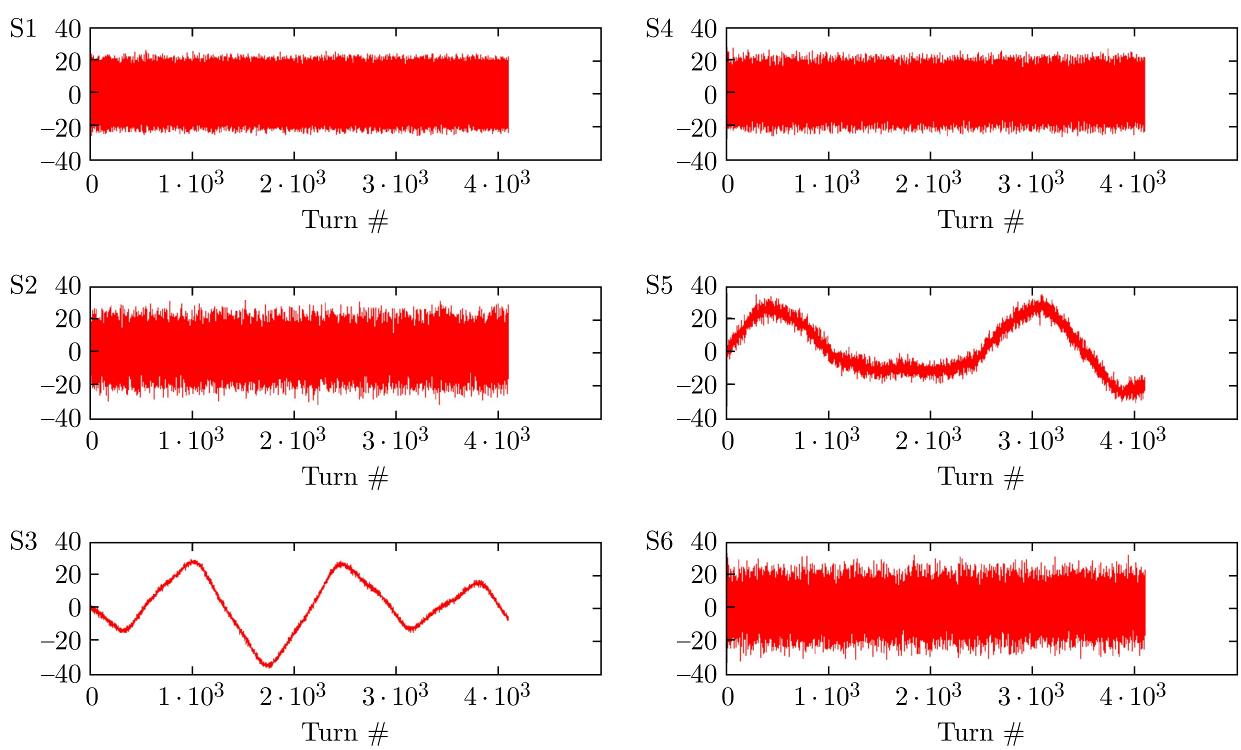

Figure 4. First six temporal modes in presence of 50-Hz crosstalk: components 1, 2 and 4, 6 present betatron motions, component 3 — synchrotron motion, component 5 — 50-Hz crosstalk.

Now we take account of the beam decoherence in a simplified model. A betatron motion excited by single kick decoheres due to betatron tune chromaticities and dependence of betatron tunes on amplitudes [18, 19]. In the NICA Booster, the main effect comes from chromaticities. In negligence of synchrotron motion and Gaussian distribution over momentum, the beam

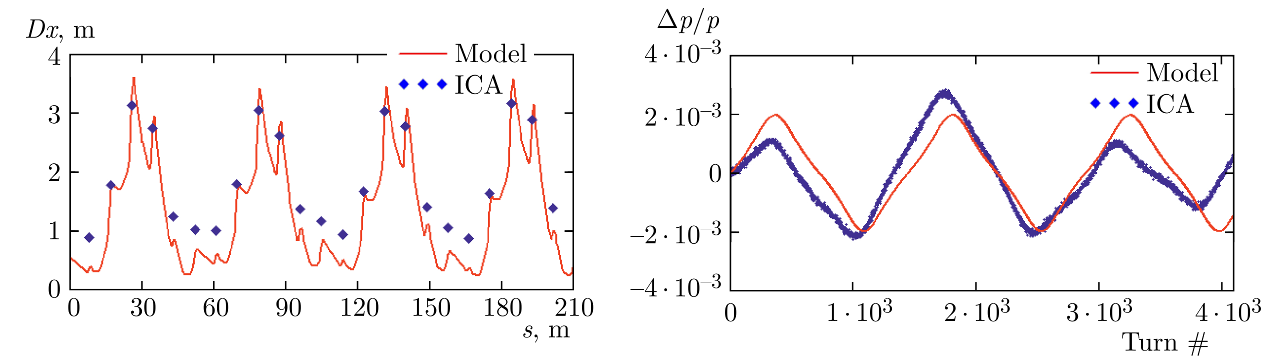

Figure 5. Results of computation of dispersion function and synchrotron motion in the system with the presence of 50-Hz crosstalk.

oscillations decohere exponentially with form-factor dependence on the turn number n as

$$F1_n = \exp\left(-2\left(\xi\frac{dp}{p}\frac{\sin\left(\pi Q_s n\right)}{Q_s}\right)^2\right),$$

where ξ is the betatron mode chromaticity, and Q_s is the synchrotron tune.

For Gaussian beam, the decoherence form factor related to the dependence of betatron tunes on the amplitude is

$$F2_n = \frac{1}{1+\theta^2}\exp\left(-\frac{A_x^2}{2\sigma_x^2}\frac{\theta^2}{1+\theta^2}\right),$$
$$\theta = 4\pi\Delta Q_x n.$$

Here A_x is the initial amplitude of beam oscillations, σ_x is the beam rms transverse size, ΔQ_x is the spread of betatron tunes in the beam. Recent measurements with injected coasting beam into the NICA Booster show that oscillations of the beam decohere completely in about 100 turns. If a large number of turns is considered (4096), then without crosstalk with frequency value close to the synchrotron frequency, the longitudinal motion and the dispersion function are restored with good accuracy ($1\cdot10^{-2}$). Consequently, it is possible to compute the betatron function with acceptable accuracy ($1\cdot10^{-1}$) and the betatron tunes with accuracy of up to $2\cdot10^{-3}$ (Figure 6).

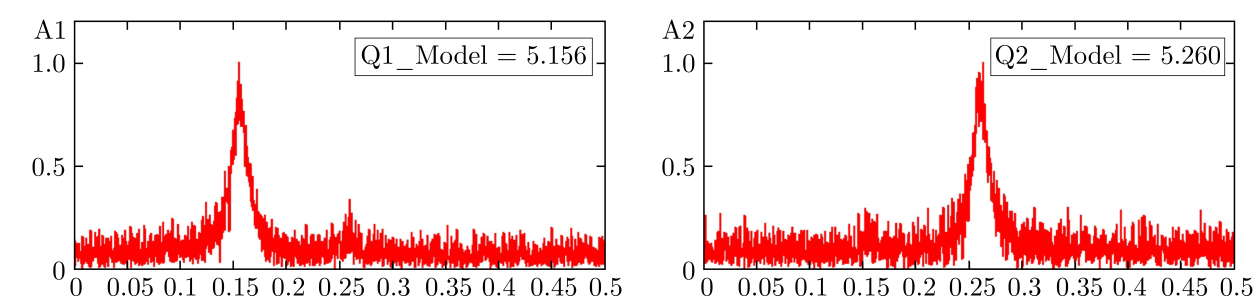

Figure 6. Results of Fourier analysis of betatron modes. Decoherence is turned ON. Values in frames are targeted tunes.

In the presence of 50-Hz crosstalk, synchrotron motion is computed with a large error. The study also showed that it is not reasonable to use the number of turns far beyond the end of coherent oscillations. Calculations with 128 turns show that the betatron function is computed with better accuracy ($3 \cdot 10^{-2}$) than for larger number of turns. However, the betatron tunes are restored with lower accuracy due to the smaller number of turns in the input matrix.

In conclusion of this section, we would like to stress that the above calculations of beta-functions and dispersions in the BPM locations are model independent but they are susceptible to misfunction of the BPMs. In particular, the above algorithms do not use the measured betatron phase advances between the BPMs. A global fitting of the ICA results with phase advances accounted into the optics model should enable one to see possible inconsistencies in BPM gains and, thus, should produce more reliable optics measurements.

4. Conclusion

For incoming measurements of linear optics in accelerators of the NICA complex, it has been necessary to produce a study based on computer simulations which would show achievable accuracy of restored optics and requirements to the measurement accuracy. None of the known methods of measurements provides absolute precision and it is quite difficult to define the degree of confidence for obtained results. Consequently, several independent methods should be used. In this study, one of the methods was tested. Simulations were based on the optics model of the NICA Booster.

Independent component analysis is a powerful tool for finding betatron functions and phases, as well as the betatron and synchrotron tunes, and dispersions. The calculations show that an application of ICA to the Booster allows one to find the betatron functions in the BPM locations with an accuracy of about 10^{-2} for moderate noises in the measurements and errors in BPM gains, even in the presence of EMI from power supplies. ICA naturally takes into account the transverse coupling if it is present in the beam optics. In normal operating conditions, it is enough to measure several hundreds of the beam turns. To find the synchrotron tunes and the dispersions and separate EMI signals with frequencies close to synchrotron frequency, several thousand turns need to be measured.

Note that the above-considered analysis is model independent. It enables finding betatron functions and betatron phases in the BPM locations without knowledge of beam optics for the rest of the machine. Although it is sensitive to errors in differential BPM responses, it supplies a redundant information (i.e. betatron amplitudes and phases) which can be used to build a reliable model of the entire machine optics. Typically, such a model is based on small corrections to quad focusing so that to match the measurements and the model. In this case, a usage of measured dispersion is highly desirable since it severely limits variations of quad focusing. Finding optimal procedures to build an actual machine optics will be a next logical step in the method development. The goal is to achieve the level of sophistication similar to the already achieved by LOCO-like algorithms which are based on the orbit response matrix. Data acquisition for the turn-by-turn measurements is much faster than for LOCO measurements, which is considerable advantage of the turn-by-turn measurements. However, from general points of view they are complimentary and a usage of both of them will significantly improve our knowledge of linear optics and its accuracy. Thus, the plan of optics measurements should include measurements of linear optics with both turn-by-turn orbit data and orbit response matrix. Also, one of goals of this comparative analysis is verification of fast methods for online optics control; for example, ICA for the turn-by-turn data or usage of just two–three orbit responses.

Acknowledgements

I wish to thank Valeri Lebedev for helpful discussions and critical review of the work.

Conflict of Interest

The author declares no conflict of interest.

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