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New classical Hall-type effect in the absence of magnetic field

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Abstract

The non-Markovian two-dimensional dynamics of charge carriers in a dissipative non-magnetic medium is studied. The possibility of observing a new classical Hall-type effect in the absence of a magnetic field is predicted.

Keywords: open quantum systems; Hall effect; non-Markovian dynamics; dissipative kernels; electric field

1. Introduction

The classical Hall effect is the occurrence of a potential difference (Hall voltage) at the edges of a sample placed in a transverse magnetic field when a current flows perpendicular to the field. The Hall voltage is proportional to the magnetic field and current strength [1, 2]. The Hall effect is related to the nature and number of charge carriers in materials. Quantitatively, the classical Hall effect can be characterized using the Hall coefficient, which is defined as the ratio of the induced electric field to the product of the current density and perpendicular magnetic field applied. In the absence of magnetic field in non-magnetic materials, a deviation of current carriers with opposite spins in different directions perpendicular to the electric field was also predicted and observed. This phenomenon, called the spin Hall effect, is related to the external spin-dependent scattering or the internal spin-orbital interaction [1, 2]. In this article, we present another possibility of observing the classical Hall-type effect in the absence of magnetic field in non-magnetic two-dimensional (2D) materials. In this case, the non-diagonal dissipative kernels effectively play the same role as the magnetic field. Note that 2D materials have remained an important research topic in the field of solid state physics and their applications for decades.

The paper is organized as follows. In Subsection 2.1, we give the Hamiltonian of 2D system (the quantum particle plus the heat bath) in the external magnetic and electric fields and derive the non-Markovian Langevin equations for the quantum particle which describes the center of mass of the charge carriers. We consider the linear coupling between the quantum particle and

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the heat bath through particle–phonon interaction. Note that the quantum Langevin approach or density matrix formalism is widely applied to find the effects of fluctuations and dissipations in the macroscopic systems [3–7]. In Subsection 2.2, the solution of the Langevin equations is presented. The analytical expressions for the time-dependent transport coefficients and components of the electric field are derived in Subsection 2.3. As shown, the cross component of the electric field (which is absent at the initial time) appears due to the dissipative effect. In Section 3, the main conclusions are given.

2. Charged carriers in external magnetic field and heat bath

2.1. Non-Markovian quantum Langevin equations

In order to investigate the influence of external fields on the dynamics of an open quantum system, we consider the motion of a charged particle (collective subsystem) with effective mass tensor (m_x, m_y, 0) and positive charge e in the neutral bosonic heat bath in the presence of the perpendicular axisymmetric magnetic field along the z axis and the time-dependent electric field acting in the xy plane. In the case of linear coupling in coordinates between the particle and the heat bath, the total Hamiltonian of the collective subsystem plus the heat bath is as follows:

$$H = \frac{1}{2m_x}(p_x - eA_x(x,y))^2 + \frac{1}{2m_y}(p_y - eA_y(x,y))^2 + exE_x(t) + \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu + \\ + \sum_\nu (\alpha_\nu x + g_\nu y)(b_\nu^+ + b_\nu) + \sum_\nu \frac{1}{\hbar\omega_\nu}(\alpha_\nu x + g_\nu y)^2, \quad (1)$$

where $\mathbf{q} = (x, y, 0)$ is the collective coordinate of a charged particle and $\mathbf{p} = (p_x, p_y, 0)$ its canonically conjugated momentum, $\mathbf{A} = (-\frac{1}{2}yB, \frac{1}{2}xB, 0)$ is the vector potential of the perpendicular axisymmetric magnetic field with the strength $B = |\mathbf{B}|$, and $\mathbf{E}(t) = (E_x(t), 0, 0)$ is the time-dependent electric field acting along the x axis. The heat bath is an assembly of noninteracting harmonic oscillators with frequencies ω_ν. The coupling to the heat bath is linear in the phonon creation b_ν^+ and annihilation b_ν operators and corresponds to the energy being transferred to and from the thermal reservoir by absorption or emission of bath quanta [3–7]. The strength of interaction of the heat bath with the collective subsystem is defined by the coupling parameters α_ν and g_ν. The last term in Eq. (1), known as the counter-term, is necessary to cancel a renormalization of the free-particle Hamiltonian induced by the interaction with the heat bath [3, 5]. This renormalization should be avoided because it creates an external 2D potential which breaks the translational invariance. As seen, Eq. (1) is set in the classical Hall geometry.

For convenience, we introduce the new definitions for momenta

$$\pi_x = p_x + \frac{1}{2}m_x\omega_{cx}y, \quad \pi_y = p_y - \frac{1}{2}m_y\omega_{cy}x, \quad (2)$$

where $\omega_{cx} = eB/m_x$, $\omega_{cy} = eB/m_y$, and $\omega_c = \sqrt{\omega_{cx}\omega_{cy}} = \frac{eB}{\sqrt{m_x m_y}}$ is the cyclotron frequency. Therefore, the Hamiltonian (1) is transformed into the form

$$H = \frac{\pi_x^2}{2m_x} + \frac{\pi_y^2}{2m_y} + exE_x(t) + \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu + \\ + \sum_\nu (\alpha_\nu x + g_\nu y)(b_\nu^+ + b_\nu) + \sum_\nu \frac{1}{\hbar\omega_\nu}(\alpha_\nu x + g_\nu y)^2. \quad (3)$$

The system of Heisenberg equations for the collective operators x, y, π_x, π_y and the bath phonon operators b_ν, b_ν^+ is obtained by commuting them with H:

$$\begin{aligned}
\dot{x}(t) &= \frac{\pi_x(t)}{m_x}, \\
\dot{y}(t) &= \frac{\pi_y(t)}{m_y}, \\
\dot{\pi}_x(t) &= \pi_y(t)\omega_{cy} - eE_x(t) - \sum_\nu \alpha_\nu (b_\nu^+(t) + b_\nu(t)) - 2\sum_\nu \frac{\alpha_\nu}{\hbar\omega_\nu}\left(\alpha_\nu x(t) + g_\nu y(t)\right), \\
\dot{\pi}_y(t) &= -\pi_x(t)\omega_{cx} - \sum_\nu g_\nu (b_\nu^+(t) + b_\nu(t)) - 2\sum_\nu \frac{g_\nu}{\hbar\omega_\nu}\left(\alpha_\nu x(t) + g_\nu y(t)\right),
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\dot{b}_\nu^+(t) &= i\omega_\nu b_\nu^+(t) + \frac{i}{\hbar}(\alpha_\nu x(t) + g_\nu y(t)), \\
\dot{b}_\nu(t) &= -i\omega_\nu b_\nu(t) - \frac{i}{\hbar}(\alpha_\nu x(t) + g_\nu y(t)).
\end{aligned} \tag{5}$$

The solutions of Eqs. (5) are

$$\begin{aligned}
b_\nu^+(t) &= f_\nu^+(t) - \frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} + \frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{x}(\tau) e^{i\omega_\nu(t-\tau)} + \frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{y}(\tau) e^{i\omega_\nu(t-\tau)}, \\
b_\nu(t) &= f_\nu(t) - \frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} + \frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{x}(\tau) e^{-i\omega_\nu(t-\tau)} + \frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{y}(\tau) e^{-i\omega_\nu(t-\tau)},
\end{aligned} \tag{6}$$

where

$$f_\nu(t) = \left[b_\nu(0) + \frac{\alpha_\nu x(0) + g_\nu y(0)}{\hbar\omega_\nu}\right] e^{-i\omega_\nu t}. \tag{7}$$

Substituting (6) into (4), we eliminate the bath variables from the equations of motion for the collective subsystem and obtain the system of integro-differential stochastic dissipative equations:

$$\begin{aligned}
\dot{x}(t) &= \frac{\pi_x(t)}{m_x}, \\
\dot{y}(t) &= \frac{\pi_y(t)}{m_y}, \\
\dot{\pi}_x(t) &= \pi_y(t)\omega_{cy} - eE_x(t) - \frac{1}{m_x}\int_0^t d\tau K_\alpha(t-\tau)\pi_x(\tau) - \frac{1}{m_y}\int_0^t d\tau K_{\alpha g}(t-\tau)\pi_y(\tau) + F_\alpha(t), \\
\dot{\pi}_y(t) &= -\pi_x(t)\omega_{cx} - \frac{1}{m_y}\int_0^t d\tau K_g(t-\tau)\pi_y(\tau) - \frac{1}{m_x}\int_0^t d\tau K_{g\alpha}(t-\tau)\pi_x(\tau) + F_g(t),
\end{aligned} \tag{8}$$

where

$$K_\alpha(t-\tau)=\sum_\nu \frac{2\alpha_\nu^2}{\hbar\omega_\nu}\cos\left[\omega_\nu(t-\tau)\right], \quad K_g(t-\tau)=\sum_\nu \frac{2g_\nu^2}{\hbar\omega_\nu}\cos\left[\omega_\nu(t-\tau)\right],$$
$$K_{\alpha g}(t-\tau)=K_{g\alpha}(t-\tau)=\sum_\nu \frac{2\alpha_\nu g_\nu}{\hbar\omega_\nu}\cos\left[\omega_\nu(t-\tau)\right] \tag{9}$$

are the dissipative kernels and

$$F_\alpha(t)=-\sum_\nu F_\alpha^\nu(t)=-\sum_\nu \alpha_\nu(f_\nu^+(t)+f_\nu(t)),$$
$$F_g(t)=-\sum_\nu F_g^\nu(t)=-\sum_\nu g_\nu(f_\nu^+(t)+f_\nu(t)) \tag{10}$$

are the random forces. The presence of the integral parts in Eqs. (8) indicates non-Markovian dynamics of the collective subsystem. The dissipative kernels have the form of memory functions since they make the equations of motion at time t dependent on the values of $\dot{x}$ and $\dot{y}$ at previous times.

The random force operators $F_\alpha^\nu(t)$ and $F_g^\nu(t)$ are identified as fluctuations due to the uncertainty of initial conditions for the bath operators. We consider an ensemble of initial states in which the operators of the collective subsystem are fixed at the values $x(0)$ and $y(0)$, and the initial bath operators are drawn from an ensemble that is canonical relative to the collective subsystem [4, 6]. The initial distribution is then the conditional density matrix $\rho_0(\{b_\nu^+(0),b_\nu(0)\}|\mathbf{q}(0))=\exp\left(-\sum_\nu \hbar\omega_\nu[b_\nu^+ + \frac{\alpha_\nu x+g_\nu y}{\hbar\omega_\nu}][b_\nu + \frac{\alpha_\nu x+g_\nu y}{\hbar\omega_\nu}]/k_BT_0\right)/Z(k_BT_0)$, where $Z(k_BT_0)$ is a conditional partition function. In an ensemble of initial states for the bath operators, the fluctuations $F_\alpha^\nu(t)$ and $F_g^\nu(t)$ have Gaussian distributions with

$$\langle\langle F_\alpha^\nu(t)\rangle\rangle \;=\; \langle\langle F_g^\nu(t)\rangle\rangle \;=\; 0, \tag{11}$$

where the symbol $\langle\langle\ldots\rangle\rangle$ denotes the averaging over the bath. The temperature T_0 of the heat bath is included in the analysis through the distribution of initial conditions. The Bose–Einstein statistics is employed for the heat bath:

$$\begin{aligned}\langle\langle f_\nu^+(t)f_{\nu'}^+(t')\rangle\rangle &= \langle\langle f_\nu(t)f_{\nu'}(t')\rangle\rangle \;=\; 0,\\ \langle\langle f_\nu^+(t)f_{\nu'}(t')\rangle\rangle &= \delta_{\nu,\nu'}n_\nu e^{i\omega_\nu(t-t')},\\ \langle\langle f_\nu(t)f_{\nu'}^+(t')\rangle\rangle &= \delta_{\nu,\nu'}(n_\nu+1)e^{-i\omega_\nu(t-t')},\end{aligned} \tag{12}$$

where $n_\nu=[\exp(\hbar\omega_\nu/(k_BT_0))-1]^{-1}$ are the occupation numbers for phonons. By employing (12), the quantum fluctuation–dissipation relations are obtained:

$$\sum_\nu \varphi_{\alpha\alpha}^\nu(t,t')\frac{\tanh[\frac{\hbar\omega_\nu}{2k_BT_0}]}{\hbar\omega_\nu} \;=\; K_\alpha(t-t'), \quad \sum_\nu \varphi_{gg}^\nu(t,t')\frac{\tanh[\frac{\hbar\omega_\nu}{2k_BT_0}]}{\hbar\omega_\nu} \;=\; K_g(t-t'),$$
$$\sum_\nu \varphi_{\alpha g}^\nu(t,t')\frac{\tanh[\frac{\hbar\omega_\nu}{2k_BT_0}]}{\hbar\omega_\nu} \;=\; K_{\alpha g}(t-t'),$$

where

$$\varphi^{\nu}_{\alpha\alpha}(t,t') = 2\alpha_\nu^2[2n_\nu+1]\cos(\omega_\nu[t-t']), \quad \varphi^{\nu}_{gg}(t,t') = 2g_\nu^2[2n_\nu+1]\cos(\omega_\nu[t-t']),$$
$$\varphi^{\nu}_{\alpha g}(t,t') = 2\alpha_\nu g_\nu[2n_\nu+1]\cos(\omega_\nu[t-t'])$$

are the symmetrized correlation functions $\varphi^{\nu}_{kk'}(t,t') = \langle\langle F^{\nu}_k(t)F^{\nu}_{k'}(t') + F^{\nu}_{k'}(t')F^{\nu}_k(t)\rangle\rangle$, $k,k' = \alpha, g$. The quantum fluctuation–dissipation relations are reduced to the classical ones in the high-temperature limit (or $\hbar \to 0$): $\sum_\nu \varphi^{\nu}_{\alpha\alpha}(t,t') = 2k_BT_0K_\alpha(t-t')$, $\sum_\nu \varphi^{\nu}_{gg}(t,t') = 2k_BT_0K_g(t-t')$, and $\sum_\nu \varphi^{\nu}_{\alpha g}(t,t') = 2k_BT_0K_{\alpha g}(t-t')$.

2.2. Solution of Langevin equations

The Laplace transform $\hat{L}$ of Eqs. (8) leads to the system of linear equations:

$$\begin{aligned} x(s)s &= x(0) + \frac{\pi_x(s)}{m_x}, \\ y(s)s &= y(0) + \frac{\pi_y(s)}{m_y}, \\ \pi_x(s)s &= \pi_x(0) + \omega_{cy}\pi_y(s) - eE_x(s) - K_\alpha(s)\frac{\pi_x(s)}{m_x} - K_{\alpha g}(s)\frac{\pi_y(s)}{m_y} + F_\alpha(s), \\ \pi_y(s)s &= \pi_y(0) - \omega_{cx}\pi_x(s) - K_g(s)\frac{\pi_y(s)}{m_y} - K_{\alpha g}(s)\frac{\pi_x(s)}{m_x} + F_g(s). \end{aligned} \tag{13}$$

Here, $K_\alpha(s)$, $K_g(s)$, $K_{\alpha g}(s)$, and $F_\alpha(s)$, $F_g(s)$ are the Laplace transforms of the dissipative kernels and random forces, respectively. The system of Eqs. (13) is easy to solve by performing the inverse Laplace transform $\hat{L}^{-1}$ and using the residue theorem and the roots of the determinant

$$D(s) = s^2 + \omega_{cx}\omega_{cy} + s\frac{K_\alpha(s)}{m_x} + s\frac{K_g(s)}{m_y} + \frac{1}{m_xm_y}\left[K_\alpha(s)K_g(s) - K^2_{\alpha g}(s)\right] = 0. \tag{14}$$

Finally, the explicit solutions for the originals are

$$\begin{aligned} x(t) &= x(0) + A_1(t)\pi_x(0) + A_2(t)\pi_y(0) + I_x(t) + I'_x(t) - I_{ex}(t), \\ y(t) &= y(0) + B_1(t)\pi_x(0) + B_2(t)\pi_y(0) + I_y(t) + I'_y(t) - I_{ey}(t), \\ \pi_x(t) &= C_1(t)\pi_x(0) + C_2(t)\pi_y(0) + I_{\pi_x}(t) + I'_{\pi_x}(t) - I_{e\pi_x}(t), \\ \pi_y(t) &= D_1(t)\pi_x(0) + D_2(t)\pi_y(0) + I_{\pi_y}(t) + I'_{\pi_y}(t) - I_{e\pi_y}(t). \end{aligned} \tag{15}$$

In Eqs. (15),

$$I_x(t) = \int_0^t A_1(\tau)F_\alpha(t-\tau)d\tau, \qquad I'_x(t) = \int_0^t A_2(\tau)F_g(t-\tau)d\tau,$$

$$I_{ex}(t) = e\int_0^t A_1(\tau)E_x(t-\tau)d\tau,$$

$$I_y(t) = \int_0^t B_1(\tau)F_\alpha(t-\tau)d\tau, \qquad I'_y(t) = \int_0^t B_2(\tau)F_g(t-\tau)d\tau,$$

$$I_{ey}(t) = e\int_0^t B_1(\tau)E_x(t-\tau)d\tau,$$

$$I_{\pi_x}(t) = \int_0^t C_1(\tau)F_\alpha(t-\tau)d\tau, \qquad I'_{\pi_x}(t) = \int_0^t C_2(\tau)F_g(t-\tau)d\tau,$$

$$I_{e\pi_x}(t) = e\int_0^t C_1(\tau)E_x(t-\tau)d\tau,$$

$$I_{\pi_y}(t) = \int_0^t D_1(\tau)F_\alpha(t-\tau)d\tau, \qquad I'_{\pi_y}(t) = \int_0^t D_2(\tau)F_g(t-\tau)d\tau,$$

$$I_{e\pi_y}(t) = e\int_0^t D_1(\tau)E_x(t-\tau)d\tau,$$

and

$$\begin{aligned}
A_1(t) &= \hat{L}^{-1}\left[\frac{m_y s + K_g(s)}{m_x m_y s D(s)}\right] = B_2(t)|_{x,\alpha\leftrightarrow y,g}, \quad A_2(t) = \hat{L}^{-1}\left[\frac{m_y\omega_{cy} - K_{\alpha g}(s)}{m_x m_y s D(s)}\right], \\
B_1(t) &= \hat{L}^{-1}\left[-\frac{m_x\omega_{cx} + K_{\alpha g}(s)}{m_x m_y s D(s)}\right], \quad C_1(t) = \hat{L}^{-1}\left[\frac{m_y s + K_g(s)}{m_y D(s)}\right] = D_2(t)|_{x,\alpha\leftrightarrow y,g}, \\
C_2(t) &= \hat{L}^{-1}\left[\frac{m_y\omega_{cy} - K_{\alpha g}(s)}{m_y D(s)}\right], \quad D_1(t) = \hat{L}^{-1}\left[-\frac{m_x\omega_{cx} + K_{\alpha g}(s)}{m_x D(s)}\right]
\end{aligned}$$

are the time-dependent coefficients.

We introduce the spectral density D_ω of the heat bath excitations to replace the sum over ν by the integral over frequency ω: $\sum_\nu \ldots \to \int_0^\infty d\omega D_\omega \ldots$, $\alpha_\nu \to \alpha_\omega$, $g_\nu \to g_\omega$, $\omega_\nu \to \omega$, and $n_\nu \to n_\omega$. The well-known spectral functions are [4]

$$\begin{aligned}
D_\omega \frac{\alpha_\omega^2}{\hbar\omega} &= \frac{\lambda_x m_x}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2}, \quad D_\omega \frac{g_\omega^2}{\hbar\omega} = \frac{\lambda_y m_y}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2}, \\
D_\omega \frac{\alpha_\omega g_\omega}{\hbar\omega} &= \frac{\kappa\eta}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2},
\end{aligned} \tag{16}$$

where the memory time γ^{-1} of dissipation is inverse to the phonon bandwidth of the heat bath excitations which are coupled with the collective subsystem. The coefficients

$$\begin{aligned}
\lambda_x &= \frac{1}{m_x}\int_0^\infty d\tau K_\alpha(t-\tau), \\
\lambda_y &= \frac{1}{m_y}\int_0^\infty d\tau K_g(t-\tau)
\end{aligned} \tag{17}$$

are the friction coefficients in the Markovian limit. This Ohmic dissipation with the Lorentzian cutoff (Drude dissipation) results in the dissipative kernels

$$\begin{aligned} K_\alpha(t-\tau) &= m_x\lambda_x\gamma e^{-\gamma|t-\tau|}, \quad K_g(t-\tau) \;=\; m_y\lambda_y\gamma e^{-\gamma|t-\tau|}, \\ K_{\alpha g}(t-\tau) &= \kappa\eta\gamma e^{-\gamma|t-\tau|}. \end{aligned} \tag{18}$$

Here, $\eta = \sqrt{m_x m_y \lambda_x \lambda_y}$ and $K_\alpha K_g - K_{\alpha g}^2 > 0$; i.e., $|\kappa| < 1$. So, employing (18), we obtain from Eqs. (16) the following time-dependent coefficients:

$$\begin{aligned} A_1(t) &= \frac{1}{m_x}\left(\frac{\lambda_y}{\omega_{cx}\omega_{cy}+(1-\kappa^2)\lambda_x\lambda_y} + \sum_{i=1}^{4}\frac{\beta_i(s_i+\gamma)\left(\gamma\lambda_y + s_i(s_i+\gamma)\right)e^{s_i t}}{s_i}\right) = B_2(t)|_{x\leftrightarrow y}, \\ A_2(t) &= \frac{1}{m_x m_y}\left(\frac{m_y\omega_{cy}-\kappa\eta}{\omega_{cx}\omega_{cy}+(1-\kappa^2)\lambda_x\lambda_y} + \sum_{1}^{4}\frac{\beta_i(s_i+\gamma)(m_y\omega_{cy}(s_i+\gamma)-\kappa\gamma\eta)e^{s_i t}}{s_i}\right), \\ B_1(t) &= -\frac{1}{m_x m_y}\left(\frac{m_x\omega_{cx}+\kappa\eta}{\omega_{cx}\omega_{cy}+(1-\kappa^2)\lambda_x\lambda_y} + \sum_{1}^{4}\frac{\beta_i(s_i+\gamma)(m_x\omega_{cx}(s_i+\gamma)+\kappa\gamma\eta)e^{s_i t}}{s_i}\right), \\ C_1(t) &= \sum_{i=1}^{4}\beta_i(s_i+\gamma)(\gamma\lambda_y+s_i(s_i+\gamma))e^{s_i t} = D_2(t)|_{x\leftrightarrow y}, \\ C_2(t) &= \sum_{i=1}^{4}\frac{\beta_i(s_i+\gamma)}{m_y}\left(m_y\omega_{cy}(s_i+\gamma)-\kappa\gamma\eta\right)e^{s_i t}, \\ D_1(t) &= -\sum_{i=1}^{4}\frac{\beta_i(s_i+\gamma)}{m_x}\left(m_x\omega_{cx}(s_i+\gamma)+\kappa\gamma\eta\right)e^{s_i t}, \end{aligned} \tag{19}$$

where $\beta_i = [\prod_{j\neq i}(s_i - s_j)]^{-1}$ $(i,j = 1-4)$ and s_i are the roots of the equation

$$\begin{aligned} D(s) \;&=\; (s+\gamma)\left[(s^2+\omega_{cx}\omega_{cy})(s+\gamma) + s\gamma\lambda_x\right] + \\ &\quad + \gamma\lambda_y\left[s(s+\gamma) + (1-\kappa^2)\gamma\lambda_x\right] = 0. \end{aligned} \tag{20}$$

2.3. Derivation of time-dependent transport coefficients and components of electric field

In order to determine the friction coefficients, the renormalized cyclotron frequencies and components of the electric field, we select the electric field as follows: $E_x(t) = E_{x0}e^{i\omega_e t}$, where ω_e is the frequency of the electric field. Averaging Eqs. (15) over the whole system, the heat bath plus the collective subsystem, using $\langle I_{x,y}(t)\rangle = \langle I'_{x,y}(t)\rangle = \langle I_{\pi_x,\pi_y}(t)\rangle = \langle I'_{\pi_x,\pi_y}(t)\rangle = 0$ (the symbol $\langle\ldots\rangle$ denotes averaging over the whole system), and differentiating them in t, we obtain the system of equations for the first moments

$$\begin{aligned} \langle\dot{x}(t)\rangle &= \frac{\langle\pi_x(t)\rangle}{m_x}, \quad \langle\dot{y}(t)\rangle \;=\; \frac{\langle\pi_y(t)\rangle}{m_y}, \\ \langle\dot{\pi}_x(t)\rangle &= -\lambda_{\pi_x}(t)\langle\pi_x(t)\rangle + \tilde{\omega}_{cy}(t)\langle\pi_y(t)\rangle - eE_{xx}(t), \\ \langle\dot{\pi}_y(t)\rangle &= -\lambda_{\pi_y}(t)\langle\pi_y(t)\rangle - \tilde{\omega}_{cx}(t)\langle\pi_x(t)\rangle - eE_{xy}(t) \end{aligned} \tag{21}$$

with the renormalized friction coefficients

$$\lambda_{\pi_x}(t) = -\frac{D_1(t)\dot{C}_2(t) - D_2(t)\dot{C}_1(t)}{C_2(t)D_1(t) - C_1(t)D_2(t)}, \quad \lambda_{\pi_y}(t) = -\frac{C_2(t)\dot{D}_1(t) - C_1(t)\dot{D}_2(t)}{C_2(t)D_1(t) - C_1(t)D_2(t)}, \tag{22}$$

the renormalized cyclotron frequencies

$$\tilde{\omega}_{cx}(t) = \frac{D_2(t)\dot{D}_1(t) - D_1(t)\dot{D}_2(t)}{C_2(t)D_1(t) - C_1(t)D_2(t)}, \quad \tilde{\omega}_{cy}(t) = \frac{C_2(t)\dot{C}_1(t) - C_1(t)\dot{C}_2(t)}{C_2(t)D_1(t) - C_1(t)D_2(t)}, \tag{23}$$

and the renormalized components of the electric field

$$\begin{aligned} E_{xx}(t) &= \frac{1}{e}\left[\lambda_{\pi_x}(t)I_{e\pi_x}(t) - \tilde{\omega}_{cy}(t)I_{e\pi_y}(t) + \dot{I}_{e\pi_x}(t)\right], \\ E_{xy}(t) &= \frac{1}{e}\left[\lambda_{\pi_y}(t)I_{e\pi_y}(t) + \tilde{\omega}_{cx}(t)I_{e\pi_x}(t) + \dot{I}_{e\pi_y}(t)\right]. \end{aligned} \tag{24}$$

Employing Eqs. (19), we obtain

$$\begin{aligned} E_{xx}(t) &= E_{x0}\sum_{i=1}^{4}\left(W_i[\lambda_{\pi_x}(t)a_i(t) + \dot{a}_i(t)] + V_i\tilde{\omega}_{cy}(t)a_i(t)\right), \\ E_{xy}(t) &= E_{x0}\sum_{i=1}^{4}\left(W_i\tilde{\omega}_{cx}(t)a_i(t) - V_i[\lambda_{\pi_y}(t)a_i(t) + \dot{a}_i(t)]\right). \end{aligned} \tag{25}$$

Here $\dot{C}_i(t) = dC_i(t)/dt$, $\dot{D}_i(t) = dD_i(t)/dt$, $\dot{I}_{e\pi_x}(t) = dI_{e\pi_x}(t)/dt$, $\dot{I}_{e\pi_y}(t) = dI_{e\pi_y}(t)/dt$, $\dot{I}'_{e\pi_x}(t) = dI'_{e\pi_x}(t)/dt$, $\dot{I}'_{e\pi_y}(t) = dI'_{e\pi_y}(t)/dt$, $\dot{a}_i(t) = da_i(t)/dt$, and

$$W_i = \beta_i\left(\gamma\lambda_y + s_i(s_i+\gamma)\right)(s_i+\gamma), \quad V_i = \beta_i\left(\omega_{cx}(s_i+\gamma) + \frac{\kappa\gamma\eta}{m_x}\right)(s_i+\gamma),$$
$$a_i(t) = \frac{e^{s_i t} - e^{i\omega_e t}}{s_i - i\omega_e}.$$

As seen, the dynamics is governed by the time-dependent friction coefficients (22), the renormalized cyclotron frequencies (23), and the renormalized components of the electric field (25). The external magnetic field generates a flow of charge carriers and the cross component $E_{xy}(t)$ of the electric field. This component is initially absent and appears during the non-Markovian evolution of the collective subsystem. In addition, the correlation between the random forces $F_\alpha(t)$ and $F_g(t)$ generates an electric field in the cross direction even at zero magnetic field.

In the axially symmetric case ($m_x = m_y = m$, $\omega_{cx} = \omega_{cy} = \omega_c$, and $\lambda_x = \lambda_y = \lambda$) and at $\kappa = 0$, Eq. (20) takes the following form:

$$D(s) = \left(s^2 + \omega_c^2\right)(\gamma + s)^2 + 2\gamma\lambda s(\gamma + s) + \lambda^2\gamma^2 = 0. \tag{26}$$

The roots of Eq. (26) are

$$\begin{aligned} s_1 &= -\frac{1}{2}\left(\gamma + i\omega_c - \sqrt{(\gamma - i\omega_c)^2 - 4\gamma\lambda}\right), \quad s_2 = s_1^*, \\ s_3 &= -\frac{1}{2}\left(\gamma + i\omega_c + \sqrt{(\gamma - i\omega_c)^2 - 4\gamma\lambda}\right), \quad s_4 = s_3^*. \end{aligned}$$

Expanding these roots up to the first order in λ/γ, we obtain

$$\begin{aligned} s_1 = s_2^* &= -\frac{\lambda\gamma^2}{\gamma^2 + \omega_c^2} - i\frac{\omega_c^2 + \gamma^2 + \gamma\lambda}{\gamma^2 + \omega_c^2}\omega_c, \\ s_3 = s_4^* &= -\gamma\frac{\omega_c^2 + \gamma^2 - \gamma\lambda}{\gamma^2 + \omega_c^2} + i\frac{\lambda\gamma\omega_c}{\gamma^2 + \omega_c^2}. \end{aligned} \tag{27}$$

Using the roots (27), Eqs. (22), (23) and $\omega_e = 0$ (the constant electric field), we derive the asymptotic renormalized friction coefficients and cyclotron frequencies:

$$\lambda_{\pi_x}(\infty) = \frac{\gamma\lambda(\gamma + \kappa\omega_c)}{\gamma^2 + \omega_c^2}, \qquad \lambda_{\pi_y}(\infty) = \frac{\gamma\lambda(\gamma - \kappa\omega_c)}{\gamma^2 + \omega_c^2} \tag{28}$$

and

$$\tilde{\omega}_{cx}(\infty) = \omega_c + \frac{\gamma\lambda(\omega_c + \kappa\gamma)}{\gamma^2 + \omega_c^2}, \qquad \tilde{\omega}_{cy}(\infty) = \omega_c + \frac{\gamma\lambda(\omega_c - \kappa\gamma)}{\gamma^2 + \omega_c^2}, \tag{29}$$

respectively. The asymptotic values of the renormalized components of the electric field (25) are

$$\frac{E_{xx}}{E_x} = 1 + \frac{\lambda(\gamma + \kappa\omega_c)}{(\gamma^2 + \omega_c^2)}, \qquad \frac{E_{xy}}{E_x} = \frac{\lambda(\omega_c + \kappa\gamma)}{(\gamma^2 + \omega_c^2)}.$$

As seen from these formulas, at zero magnetic field ($\omega_c = 0$) the classical Hall effect is expected to be observed due to the nonzero correlations between the random forces F_g and F_α (i.e., $\kappa \neq 0$). In the formula for E_{xy}/E_x, the non-diagonal dissipative kernels effectively play the role of magnetic field ($\tilde{\omega}_{cx,cy}(\infty) \sim \pm\kappa$). In the Markovian limit, $\lambda_{p_x}(\infty) = \lambda_x$, $\lambda_{p_y}(\infty) = \lambda_y$, $\tilde{\omega}_{cx}(\infty) = \kappa\eta/m_x$, $\tilde{\omega}_{cy}(\infty) = -\kappa\eta/m_y$, and $E_{xx} = E_x$, $E_{xy}/E_x = 0$. So, the cross electric field and, correspondingly, new classical Hall-type effect does not appear in the Markovian limit: $E_{xx} = E_x$ and $E_{xy}/E_x = 0$.

3. Conclusion

Using the non-Markovian quantum Langevin approach and taking into account the coupling of 2D charge carriers to the environment and the correlations between random forces in the collective coordinates x and y or the mixed non-diagonal dissipative kernels, we predicted the classical Hall-type effect in the 2D non-magnetic materials in the absence of external magnetic field and non-Markovian limit. Similarly, we can predict the classical Nernst–Ettingshausen-type and Righi–Leduc-type effects [1, 2] at zero magnetic field in a non-magnetic material. Thus, along with thermomagnetic and galvanomagnetic phenomena, we predict the existence of thermodissipative and galvanodissipative phenomena.

Conflicts of Interest

The authors declare no conflicts of interest.

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